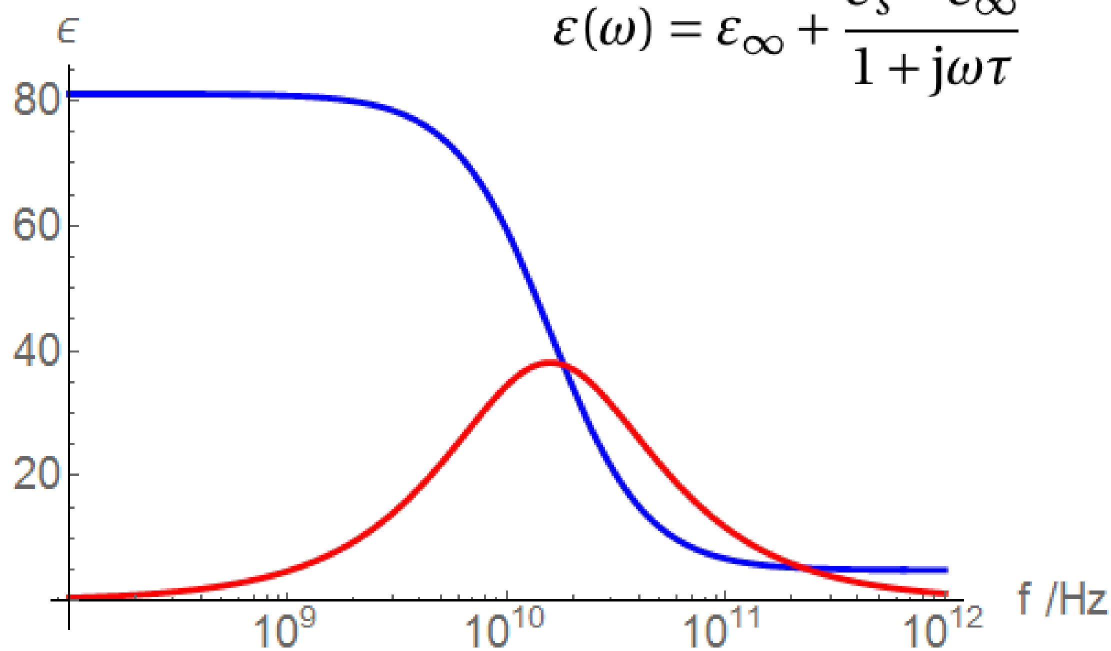
Complex permittivity of water (Debye model)

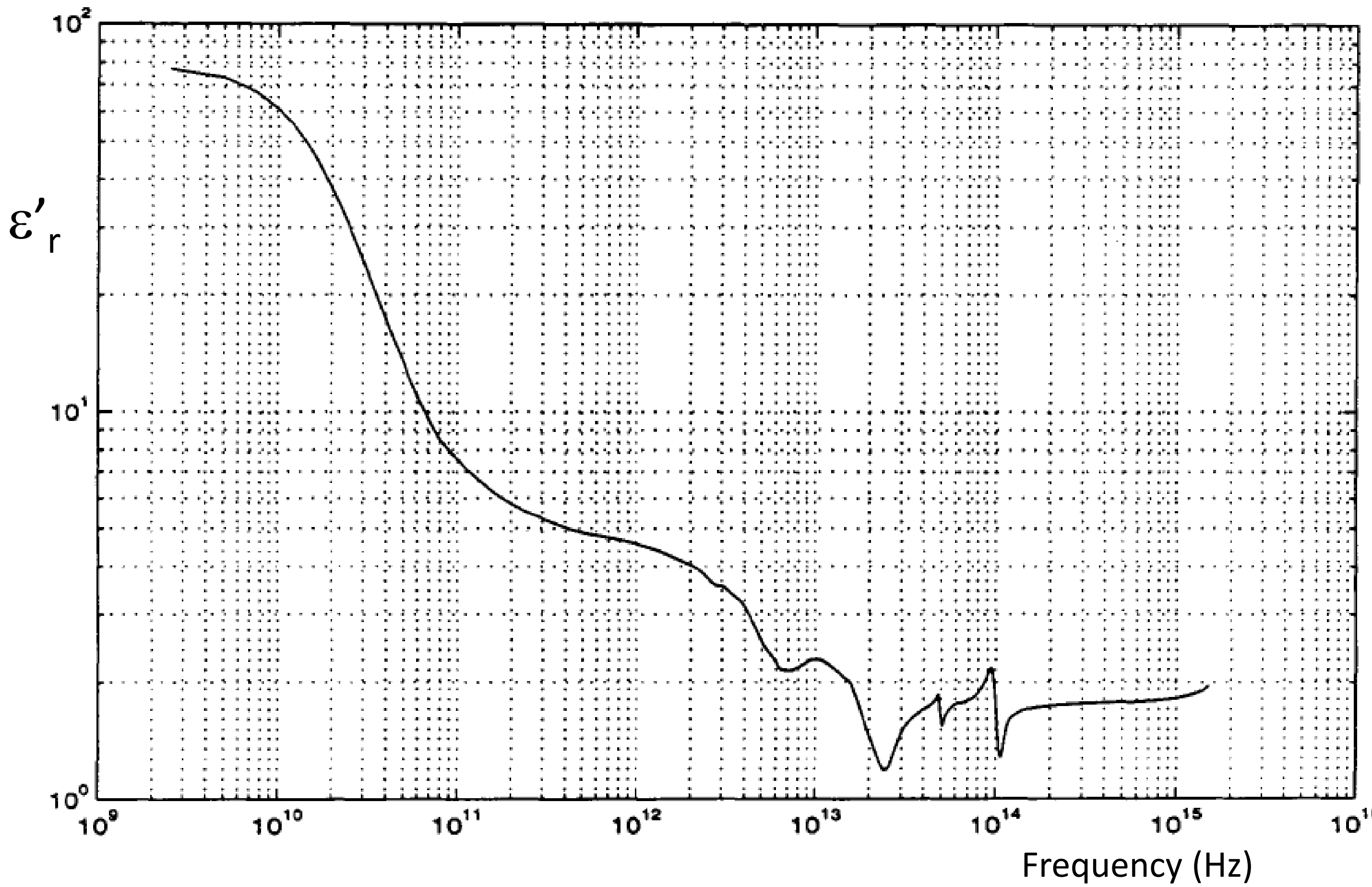
Relative permittivity values at 20°C

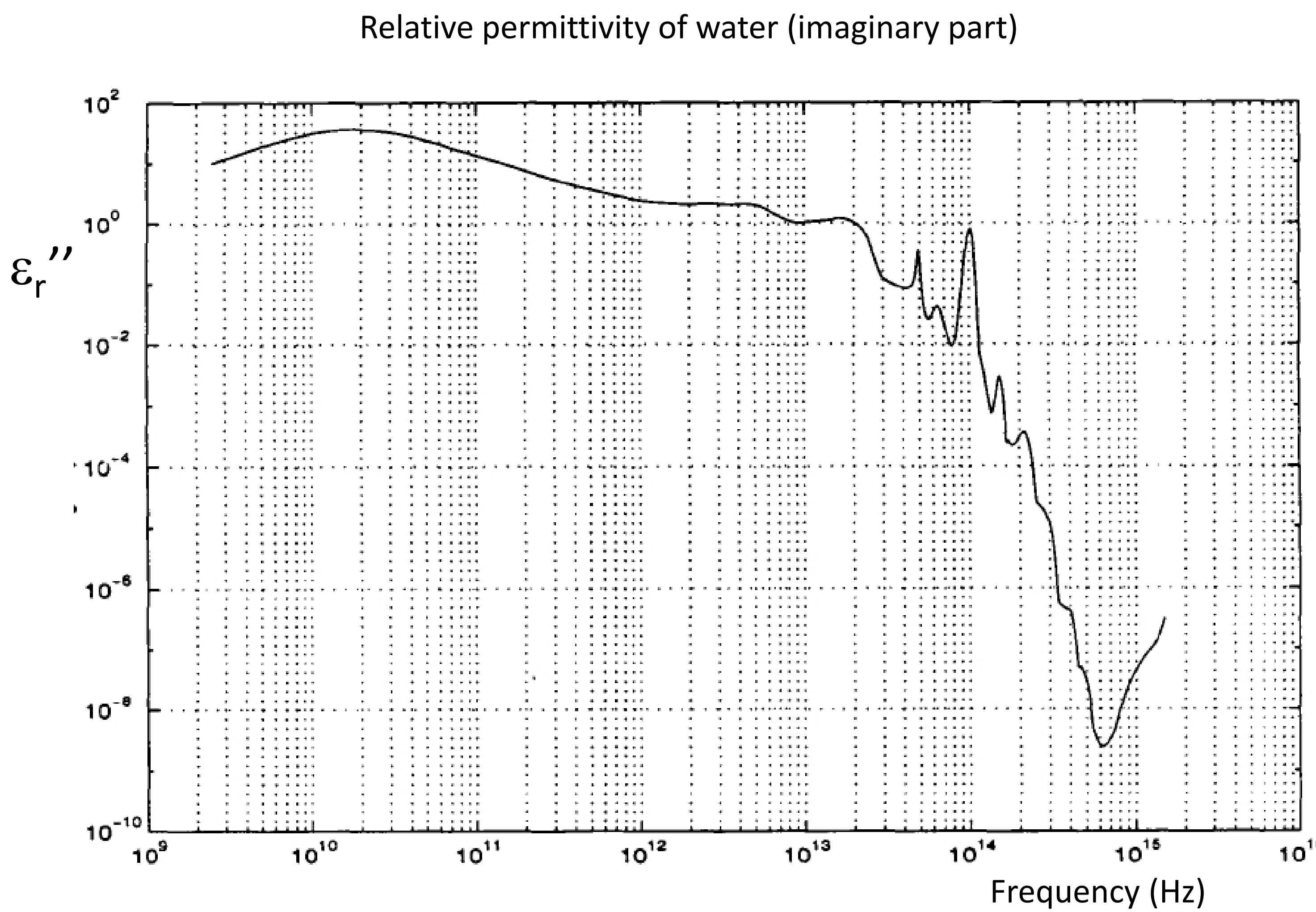
$$\varepsilon_s = 81.1$$
$$\varepsilon_\infty = 4.90$$
$$\tau = 10.1\,\text{ns}$$

$$\varepsilon(\omega) = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + j\omega\tau}$$



Relative permittivity of water (real part)





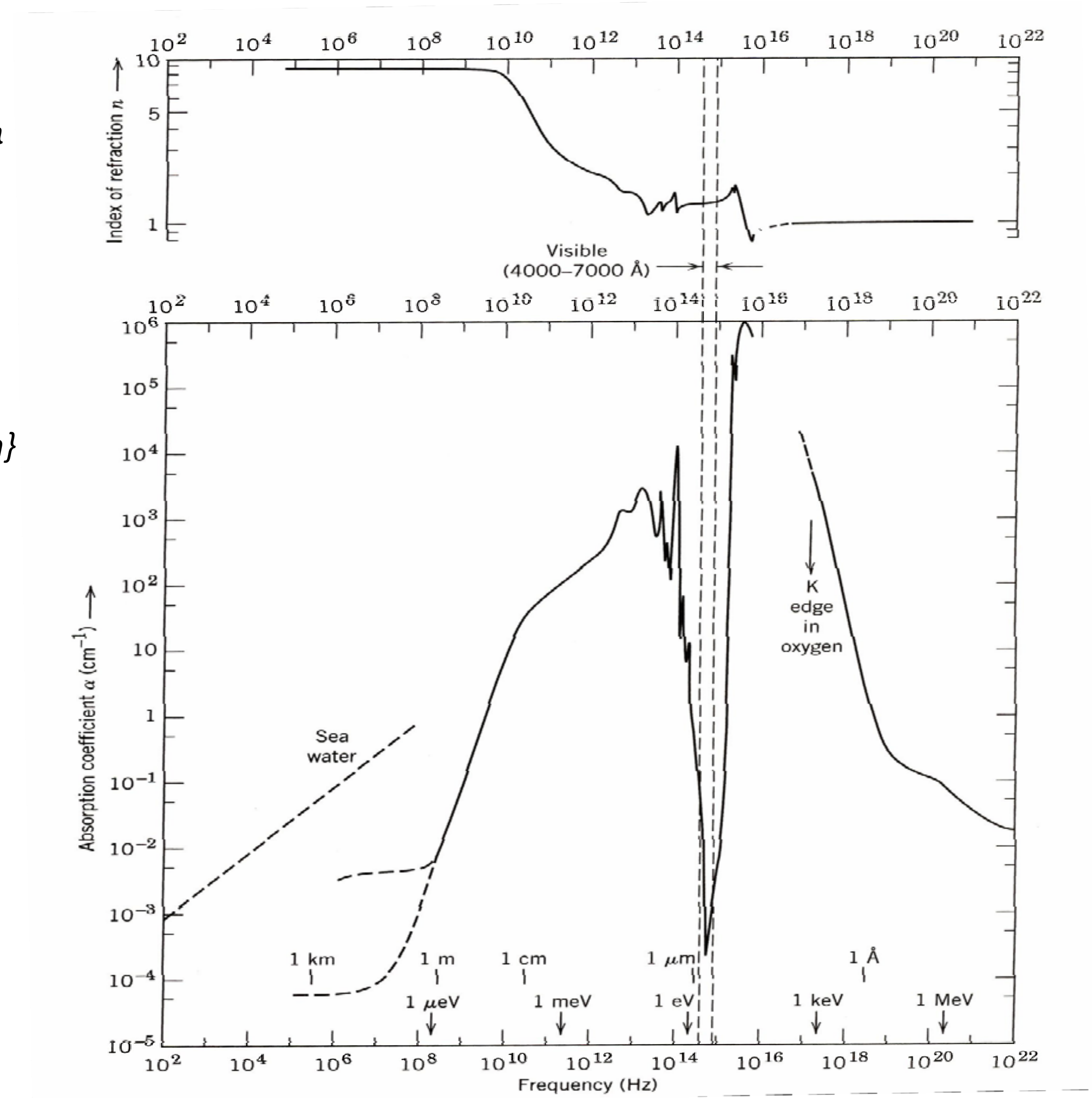
Relative permittivity of water (imaginary part)
ε_r''
10^2
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^9
10^{10}
10^{11}
10^{12}
10^{13}
10^{14}
10^{15}
10^{16}
Frequency (Hz)

Refrective index of water

$$n = \sqrt{\varepsilon_r}$$

Re{*n*}

Im{*n*}



Another dispersion model: Lorentz

$$\sin a = \frac{e^{ja} - e^{-ja}}{2j}$$

$$\chi(t) = H(t)\,\beta \sin(\omega_F t)\, e^{-\alpha t}$$

$$\chi(\omega) = \int_{-\infty}^{\infty} \chi(t) e^{-j\omega t}\, dt = \beta \int_0^{\infty} \sin(\omega_p t)\, e^{-(\alpha + j\omega)t}\, dt$$

$$= \frac{\beta}{2j} \int_0^{\infty} \left(e^{-(\alpha + j(\omega - \omega_p))t} - e^{-(\alpha + j(\omega + \omega_p))t} \right) dt$$

$$= \frac{\beta}{2j} \Bigg/_0^{\infty} \frac{e^{-(\alpha + j(\omega - \omega_F))t}}{-(\alpha + j(\omega - \omega_F))} - \frac{e^{-(\alpha + j(\omega + \omega_p))t}}{-(\alpha + j(\omega + \omega_p))}$$

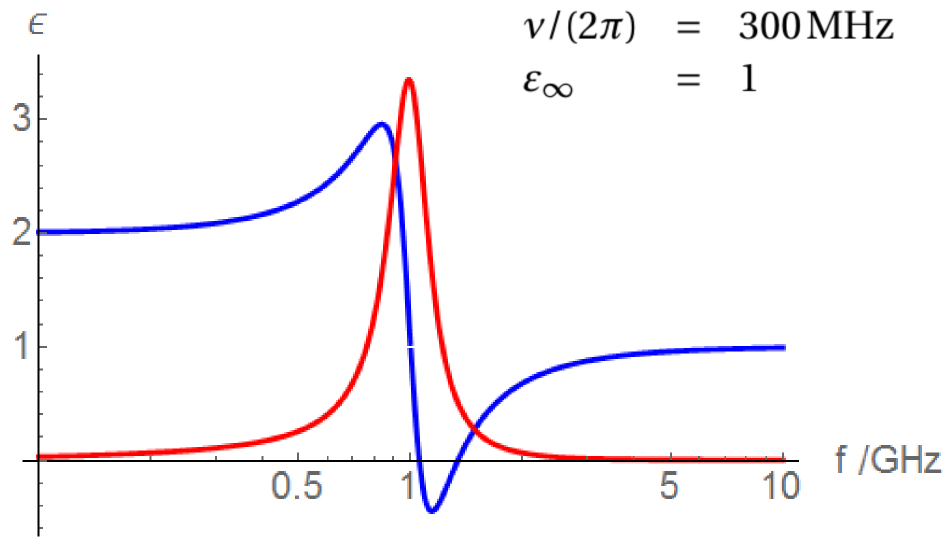
$$= \frac{\beta}{2j} \left(\frac{1}{\alpha + j(\omega - \omega_p)} - \frac{1}{\alpha + j(\omega + \omega_p)} \right)$$

$$= \frac{\beta}{2j} \cdot \frac{\alpha + j\omega + j\omega_F - \alpha - j\omega + j\omega_F}{(\alpha + j\omega)^2 - (j\omega_F)^2} = \frac{\alpha \beta}{\omega_F^2 + \alpha^2 - \omega^2 + 2j\alpha\omega}$$

Lorentz

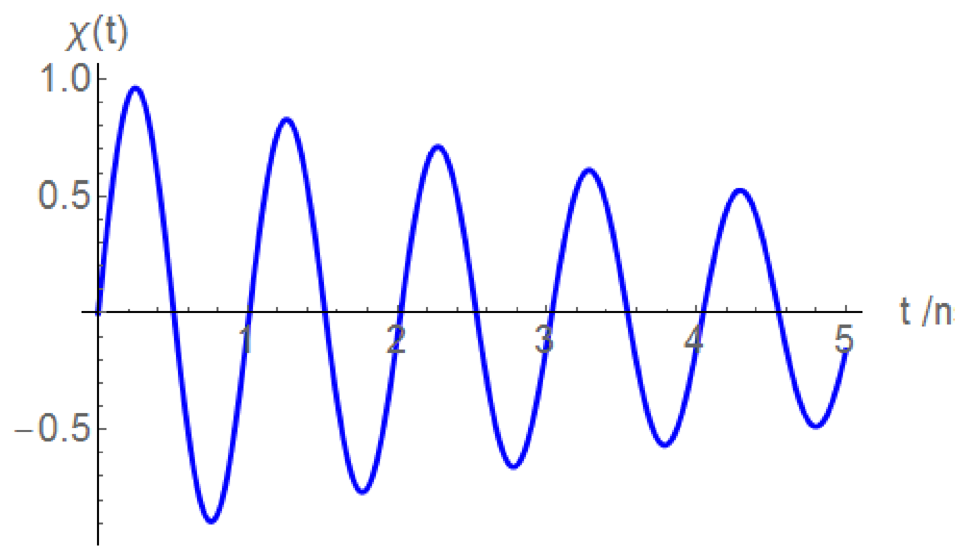
$$\varepsilon(\omega) = \varepsilon_\infty + \frac{\omega_p^2}{\omega_0^2 - \omega^2 + j\omega\nu}$$

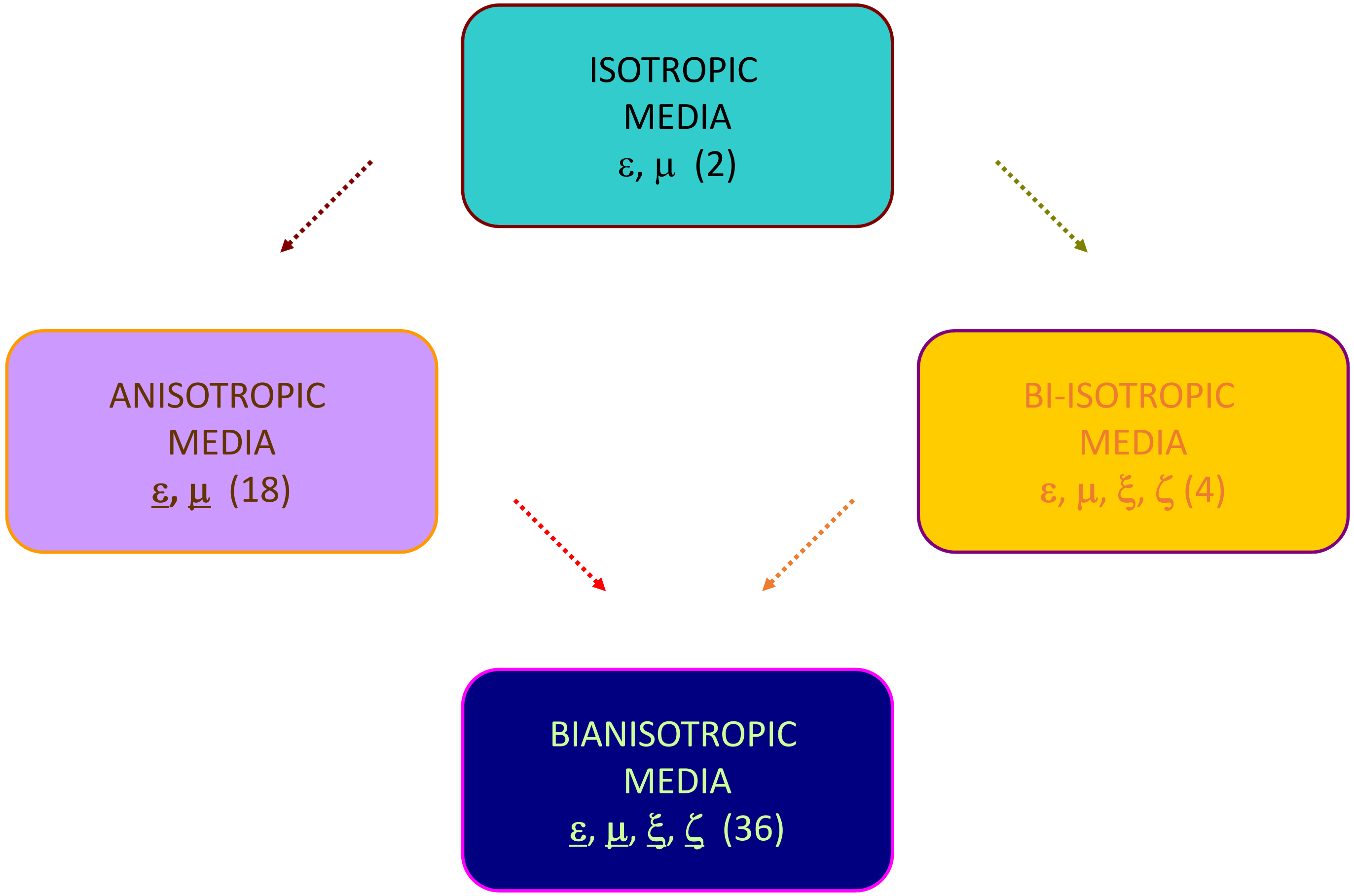
$$\begin{aligned} f_p &= 1\,\text{GHz} \\ f_0 &= 1\,\text{GHz} \\ \nu/(2\pi) &= 300\,\text{MHz} \\ \varepsilon_\infty &= 1 \end{aligned}$$

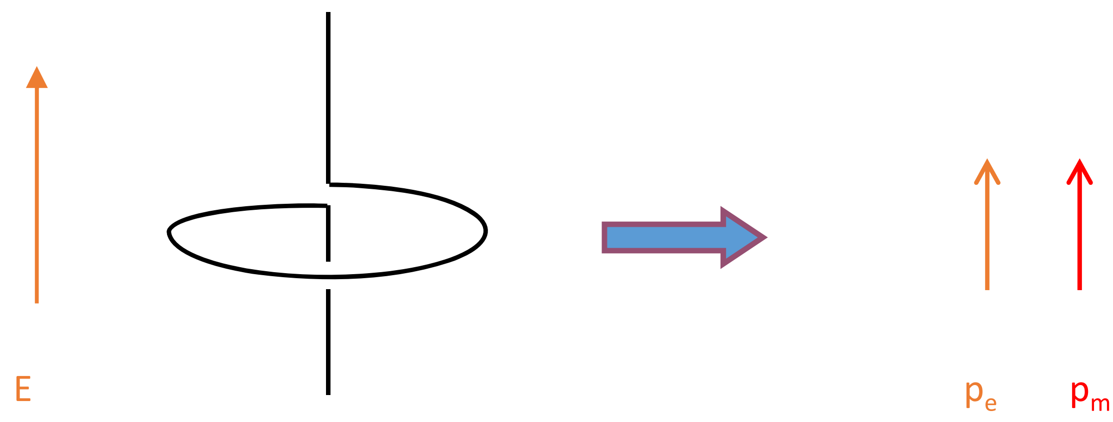


$$\chi(t) = H(t)\frac{\omega_p^2}{\omega_F}\sin(\omega_f t)e^{-\nu t/2}$$

$$\omega_F = \sqrt{\omega_o^2 - (\nu/2)^2} \approx (2\pi)\cdot 0.989\,\text{GHz}$$




ISOTROPIC MEDIA
ε, μ (2)
ANISOTROPIC MEDIA
ε, μ (18)
BI-ISOTROPIC MEDIA
ε, μ, ξ, ζ (4)
BIANISOTROPIC MEDIA
ε, μ, ξ, ζ (36)



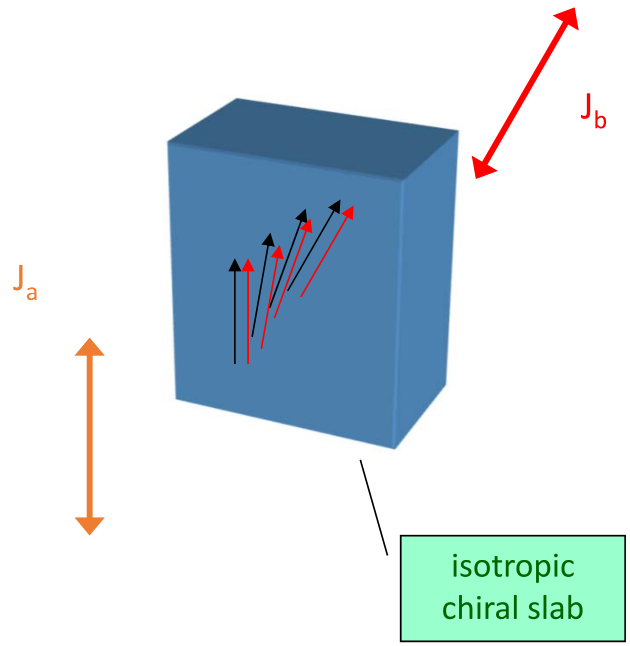
E
p_e
p_m

Pasteur (reciprocal) media

$$\begin{pmatrix} \mathbf{D} \\ \mathbf{B} \end{pmatrix} = \begin{pmatrix} \varepsilon & -j\kappa \\ j\kappa & \mu \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}$$

κ chirality parameter (Pasteur parameter)

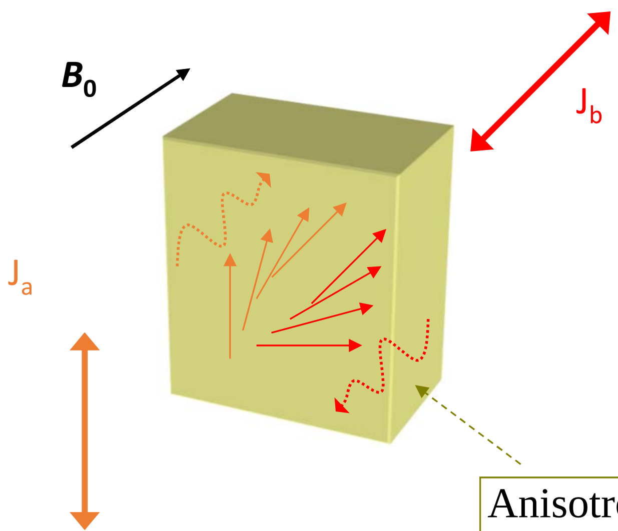
Optical activity



Pasteur medium reciprocal:

$$\int \mathrm{E}_a \cdot \mathrm{J}_b \, dV = \int \mathrm{E}_b \cdot \mathrm{J}_a \, dV$$

Faraday rotation



Magnetoplasma non - reciprocal :

$$\int \mathrm{E_a} \cdot \mathrm{J_b} \, dV \neq \int \mathrm{E_b} \cdot \mathrm{J_a} \, dV$$

Anisotropic permittivity :

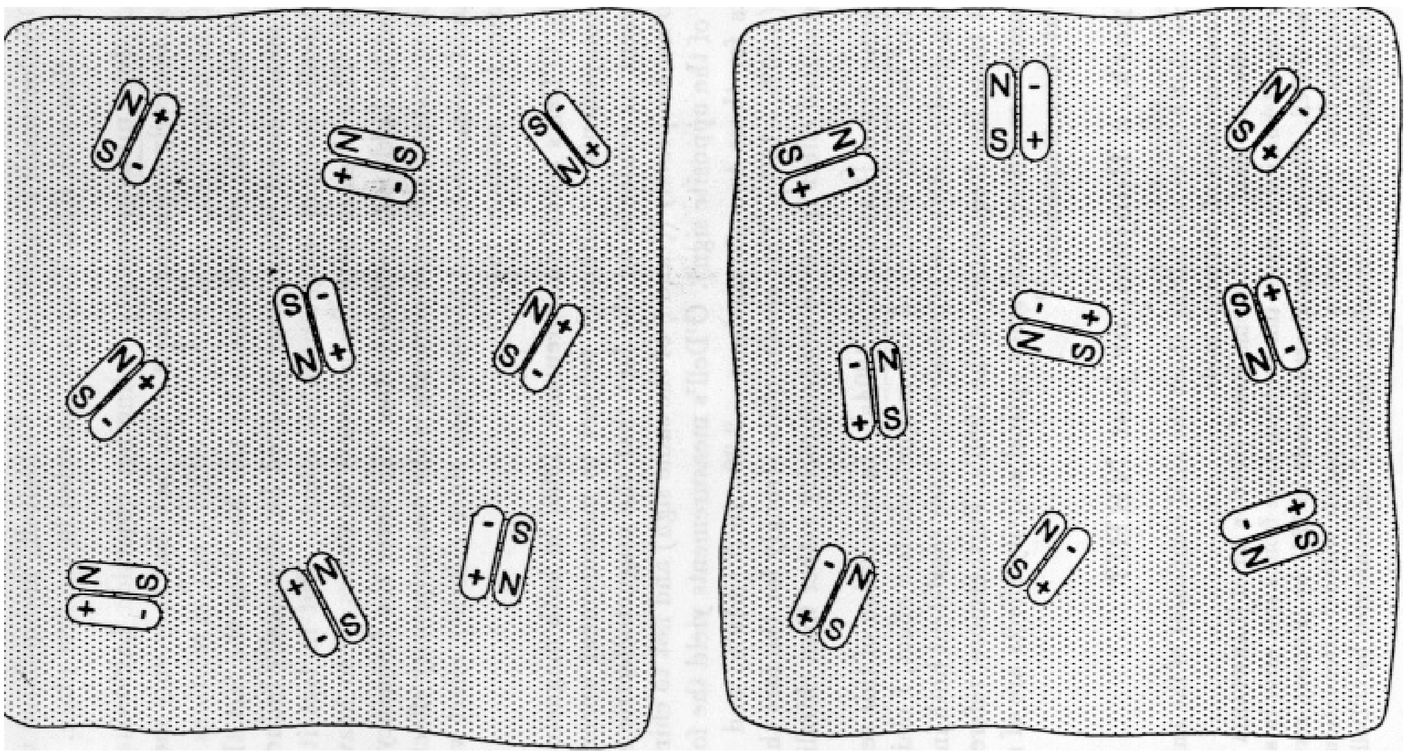
$$\varepsilon = \varepsilon_{\mathrm{symm}} + \mathrm{j}\,\mathrm{g} \times I$$

Tellegen (non-reciprocal) media

$$\begin{pmatrix} D \\ B \end{pmatrix} = \begin{pmatrix} \varepsilon & \chi \\ \chi & \mu \end{pmatrix} \begin{pmatrix} E \\ H \end{pmatrix}$$

χ non-reciprocity parameter (Tellegen parameter)

Tellegen (NRBI) material



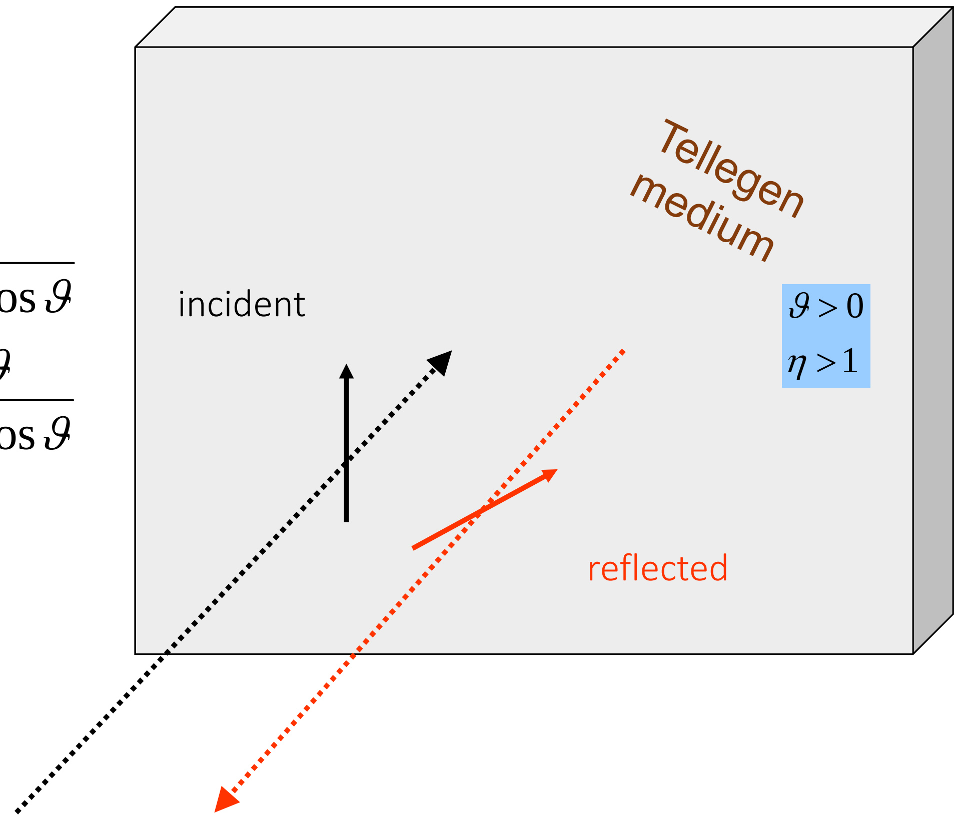
Tellegen: non-reciprocal reflection

$$R_{xx} = \frac{\eta^2 - 1}{\eta^2 + 1 + 2\eta\cos\vartheta}$$

$$R_{xy} = \frac{-2\eta\sin\vartheta}{\eta^2 + 1 + 2\eta\cos\vartheta}$$

$$\sin\vartheta = \frac{\chi}{\sqrt{\varepsilon\mu}}$$

$$\eta = \sqrt{\mu/\varepsilon}$$



Bi-isotropic media

$$\begin{pmatrix} D \\ B \end{pmatrix} = \begin{pmatrix} \varepsilon & \chi - j\kappa \\ \chi + j\kappa & \mu \end{pmatrix} \begin{pmatrix} E \\ H \end{pmatrix}$$

$\xi = \chi - j\kappa$

$\zeta = \chi + j\kappa$

κ chirality parameter (Pasteur)

χ non-reciprocity parameter (Tellegen)

Bianisotropy: the word

WAVE PROPAGATION

IN MOVING ANISOTROPIC MEDIA

ABSTRACT OF DISSERTATION

Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Electrical Engineering in the Graduate School of Syracuse University, June 1968.

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B.S.E.E., National Taiwan University, 1962
M.S.E.E., National Chiao Tung University, 1965

Bianisotropy

ABSTRACT

The primary objective of this dissertation is to introduce the concept of a bianisotropic medium and to study some wave propagation problems of recent interest which involve anisotropic media of motion. A bianisotropic medium is defined as one in which the field vectors $\bar{D}$ and $\bar{H}$ depend upon both $\bar{E}$ and $\bar{B}$ but may not be parallel to either. A moving medium appears bianisotropic to the laboratory observer even if it is isotropic in its rest frame. General transformation formulas for

from anisotropy to bianisotropy

$$D = \varepsilon \cdot E$$

dyadic
(matrix)

$$D = \varepsilon \cdot E + \xi \cdot H$$

$$B = \zeta \cdot E + \mu \cdot H$$

Constitutive relations: bi-anisotropic media

$$\begin{pmatrix} D \\ B \end{pmatrix} = \begin{pmatrix} \bar{\bar{\varepsilon}} & \bar{\bar{\xi}} \\ \bar{\bar{\zeta}} & \bar{\bar{\mu}} \end{pmatrix} \begin{pmatrix} E \\ H \end{pmatrix}$$

Bianisotropic constitutive relations

$$D = \varepsilon \cdot E + \xi \cdot H$$

$$B = \zeta \cdot E + \mu \cdot H$$

$$\xi = \chi^{\mathrm{T}} - \mathrm{j}\kappa^{\mathrm{T}}$$

$$\zeta = \chi + \mathrm{j}\kappa$$

nonreciprocity dyadic (χ)

chirality dyadic (κ)

$$\text{Lossless}: \quad \xi = \zeta^{*\mathrm{T}} \Rightarrow \chi^{\mathrm{T}} - \mathrm{j}\kappa^{\mathrm{T}} = (\chi + \mathrm{j}\kappa)^{*\mathrm{T}} \Rightarrow \chi, \kappa \text{ real}$$

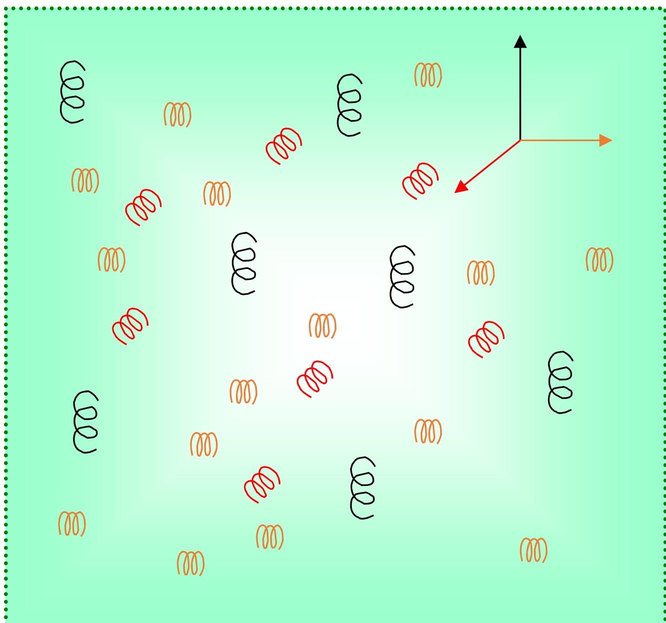
$$\text{Reciprocal}: \xi = -\zeta^{\mathrm{T}} \Rightarrow \chi^{\mathrm{T}} - \mathrm{j}\kappa^{\mathrm{T}} = -(\chi + \mathrm{j}\kappa)^{\mathrm{T}} \Rightarrow \chi = 0,\ \kappa \text{ arbitary}$$

Classification of bi-anisotropic materials

	ε	μ	κ	χ
Symmetric part: 6 parameters	(RECIPROCAL) **Dielectric crystal**	**Magnetic medium**	**Chiral medium**	$\mathbf{Cr_2O_3}$
Anti-symmetric part 3 parameters	(NON-RECIPROCAL) **Magneto-plasma**	**Biased ferrite**	**Omega medium**	**Moving medium**

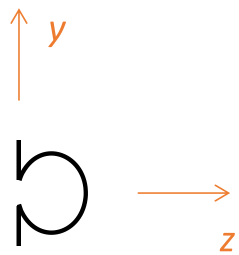
A. Sihvola, I.V. Lindell (2008), Perfect electromagnetic conductor medium, *Ann. der Physik*, ***17****(9-10), 787-802*

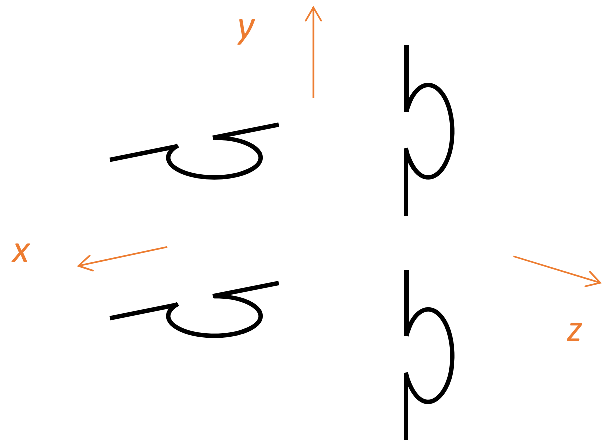
Chirality dyadic *(symmetric)*



$\kappa \, \bar{u}\bar{u} + \kappa \, \bar{u}\bar{u} + \kappa \, \bar{u}\bar{u}$

Omega medium





$$\xi = \mathrm{j}\omega \begin{pmatrix} 0 & 0 & 0 \\ \Omega & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\xi = \mathrm{j}\omega \begin{pmatrix} 0 & -\Omega & 0 \\ +\Omega & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$