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Time Series Analysis
Dummy Variables

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Introduction to Dummy Variables

Dummy variables (aka. Indicator variables or Binary variables)

Seasonality:

Example: Four (4) seasons 1, 2, 3, 4 (Quarters)

Time	Data quarter	D_1	D_2	D_3	D_4	Constant
1	1	1	0	0	0	1
2	2	0	1	0	0	1
3	3	0	0	1	0	1
4	4	0	0	0	1	1
5	1	1	0	0	0	1
6	2	0	1	0	0	1
7	3	0	0	1	0	1
8	4	0	0	0	1	1

↑

Categorical variable equivalent to 4 dummies (mutually exclusive)

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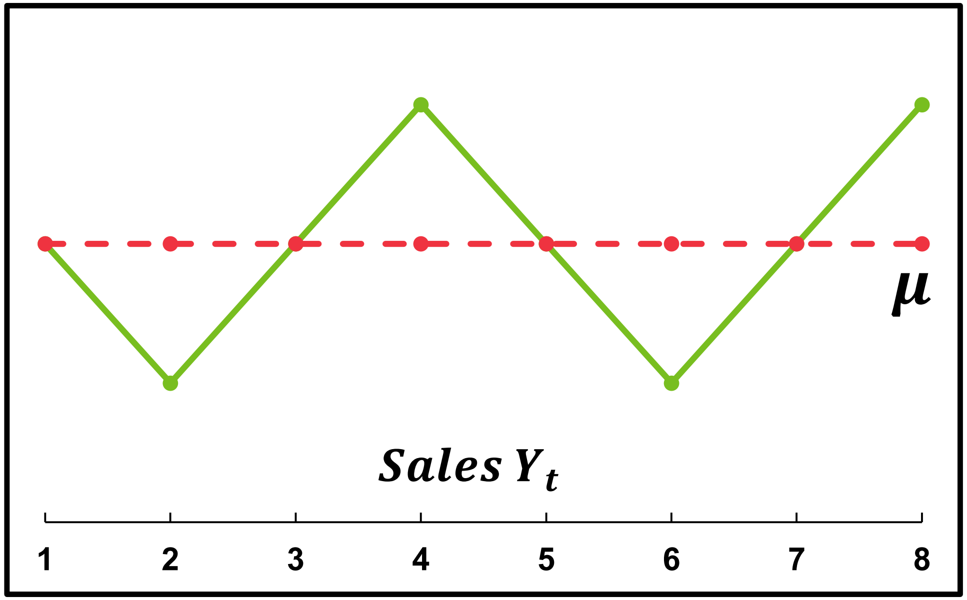

Model:

$$Y_t = \mu + \gamma_1 D_1 + \gamma_2 D_2 + \gamma_3 D_3 + \gamma_4 D_4 + u_t$$

Complete linear dependence → We cannot estimate

$$\mu = D_1 + D_2 + D_3 + D_4$$

There are two alternatives

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Alternative 1:

$$Y = \mu + \gamma_1 D_1 + \gamma_2 D_2 + \gamma_3 D_3 + u$$

- Drop one of the ***D***'s from the analysis
- Season 4 is the reference group (D_4) against which the other groups are compared
- Regression analysis table (ANOVA) gives t- and p-values compared to group 4:

$$H_0: \gamma_i = 0$$
$$H_1: \gamma_i \neq 0$$

separate individual tests for each $i = 1, 2, 3$

(Pairwise comparison between group ***i*** and 4)

Overall seasonality test: **is there seasonality in the data**:

Joint test $H_0: \gamma_1 = \gamma_2 = \gamma_3 = 0$ ← *No seasonality*

$$H_1: \gamma_1 \neq 0 \; or \; \gamma_2 \neq 0 \; or \; \gamma_3 \neq 0$$

ALWAYS DO OVERALL SEASONALITY TEST FIRST BEFORE ANY INDIVIDUAL TESTS!

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Alternative 2: Keep all the dummies, but drop μ

$$Y_t = \gamma_1 D_1 + \gamma_2 D_2 + \gamma_3 D_3 + \gamma_4 D_4 + u_t$$

No constant included

Joint test for overall seasonality (ANOVA to test means)

$$H_0: \gamma_1 = \gamma_2 = \gamma_3 = \gamma_4$$
$$H_1: \gamma_i \neq \gamma_j \; for\ some\ i \neq j$$

Possible pairwise tests; for example:

$$H_0: \gamma_1 = \gamma_2$$
$$H_0: \gamma_1 \neq \gamma_2$$

Another way to deal with seasonality

$$Y_t = \mu + \alpha \cdot \sin(\mathrm{ct})$$

$$sin(0) = 0$$

$$sin(\frac{\pi}{2}) = 1$$

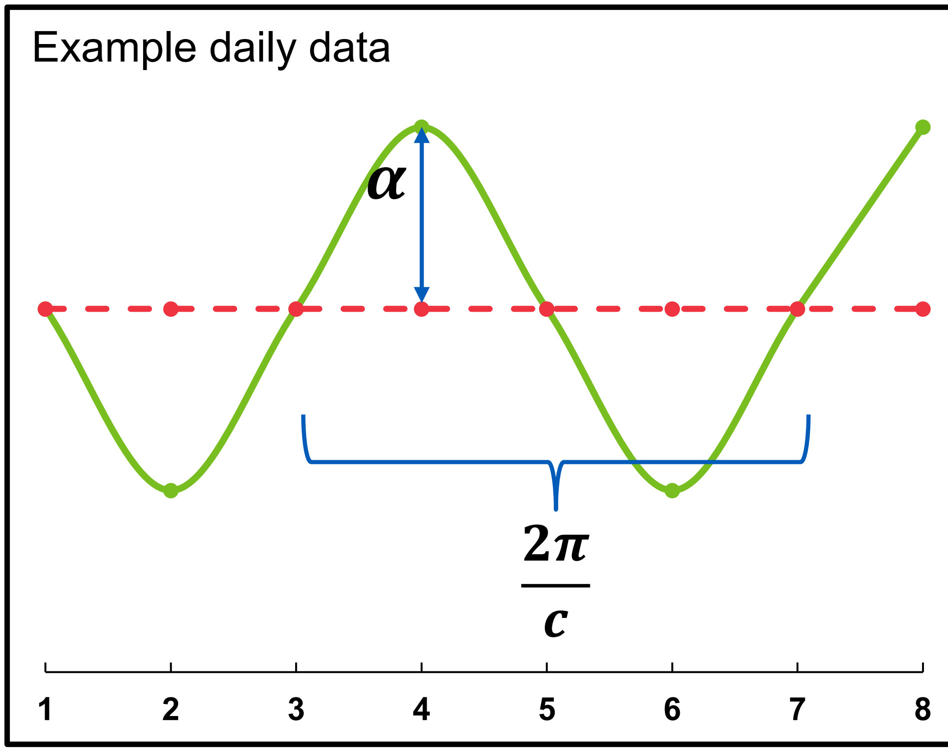
$$sin(\pi) = 0$$

$$sin\left(\frac{3\pi}{2}\right) = -1$$

$$sin(2\pi) = 0$$

Length of period:

$$\mathrm{ct} = 2\pi \rightarrow t = \frac{2\pi}{c}$$

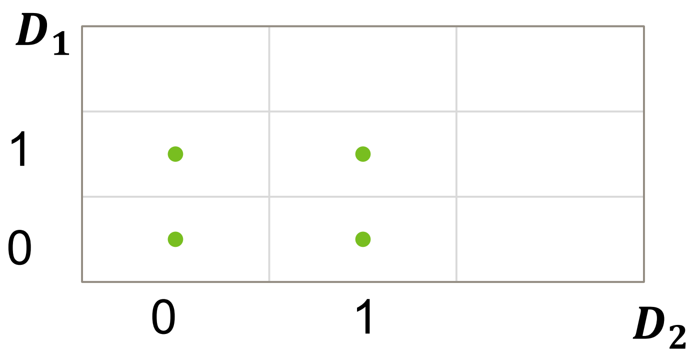


Interaction effects

Two separate dummy variables D_1, D_2, which could happen at the same time (or not). Note: these cannot be seasonal dummies, since they would be mutually exclusive

Interaction effect is $D_1 D_2$ (obtained with multiplying)

Four possibilities:



Model:

$$Y = \mu + \gamma_1 D_1 + \gamma_2 D_2 + \gamma_{12} D_1 D_2 + u$$

Main effects ($\gamma_1 D_1 + \gamma_2 D_2$) *Interaction* ($\gamma_{12} D_1 D_2$)

Note: subscript t is left out for simplicity

Interaction effects

Model:

$$Y = \mu + \underbrace{\gamma_1 D_1 + \gamma_2 D_2}_{\textit{Main effects}} + \underbrace{\gamma_{12} D_1 D_2}_{\textit{Interaction}} + u$$

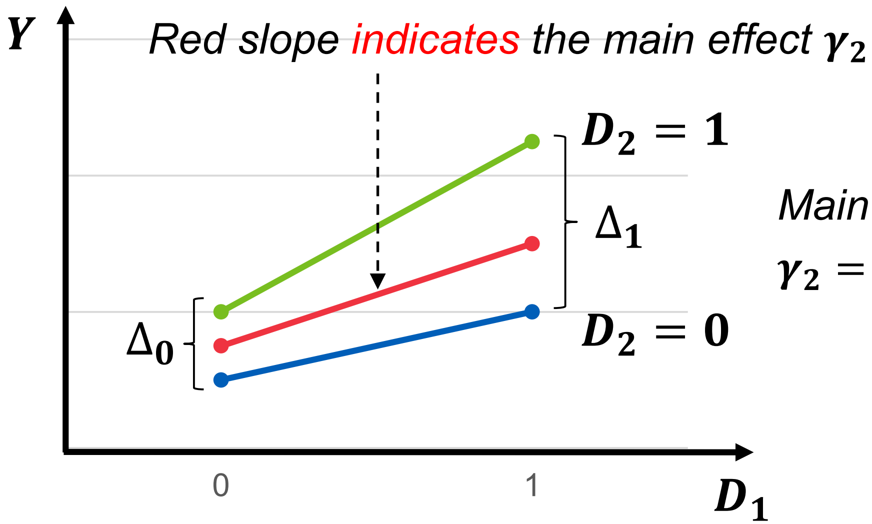
D_1 \ D_2	0	1
1	$\mu + \gamma_2 + u_t$	$\mu + \gamma_1 + \gamma_2 + \gamma_{12} + u_t$
0	$\mu + u_t$	$\mu + \gamma_1 + u_t$

Interaction effects

Test for interaction:

$$H_0: \gamma_{12} = 0$$
$$H_1: \gamma_{12} \neq 0$$

Test if the slopes are the same



Main effect
$$\gamma_2 = \frac{\Delta_1 + \Delta_0}{2}$$

Interpretation:

1. γ_{12} is the difference in slopes or alternatively
2. γ_{12} is the difference $\Delta_1 - \Delta_0$
3. If there is interaction, the effect on Y when changing D_1 from 0 to 1 is different for those with D_2=0 compared to those with D_2=1. i.e. green and blue slopes are the same