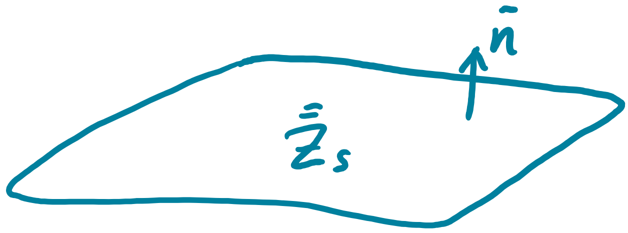
$$\begin{pmatrix} \bar{D} \\ \bar{B} \end{pmatrix} = \begin{pmatrix} \bar{\bar{\varepsilon}} & \bar{\bar{\xi}} \\ \bar{\bar{\zeta}} & \bar{\bar{\mu}} \end{pmatrix} \cdot \begin{pmatrix} \bar{E} \\ \bar{H} \end{pmatrix}$$

RECIPROCITY: $\bar{\bar{\varepsilon}}^T = \bar{\bar{\varepsilon}}, \quad \bar{\bar{\mu}} = \bar{\bar{\mu}}^T, \quad \bar{\bar{\zeta}}^T = -\bar{\bar{\xi}}$

$$\bar{\bar{\xi}} = \bar{\bar{\chi}}^T - j\bar{\bar{\kappa}}^T \qquad \bar{\bar{\zeta}} = \bar{\bar{\chi}} + j\bar{\bar{\kappa}}$$

LOSSLESS: $\bar{\bar{\varepsilon}}^T = \bar{\bar{\varepsilon}}^*, \quad \bar{\bar{\mu}}^* = \bar{\bar{\mu}}^T, \quad \bar{\bar{\zeta}}^* = \bar{\bar{\xi}}^T$

SURFACE



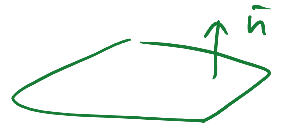
$$\bar{E}_t = \bar{\bar{Z}}_s \cdot \bar{n} \times \bar{H}_t$$

RECIPROCITY: $\bar{\bar{Z}}_s^T = \bar{\bar{Z}}_s$

LOSSLESS: $\bar{\bar{Z}}_s^* = -\bar{\bar{Z}}^T$

2D dyadic $\bar{\bar{Z}}_s$ $\qquad$ $\bar{n}\cdot\bar{\bar{Z}}_s = 0$

$\bar{\bar{Z}}_s\cdot\bar{n} = 0$



$\bar{u}, \bar{v}, \bar{n}$

$$\bar{\bar{I}}_t = \bar{\bar{I}} - \bar{n}\bar{n}$$

$$\bar{\bar{I}}_t = \bar{u}\bar{u} + \bar{v}\bar{v}$$

$$\bar{\bar{J}} = \bar{n}\times\bar{\bar{I}} = \bar{n}\times\bar{\bar{I}}_t = \bar{v}\bar{u} - \bar{u}\bar{v}$$

$$\bar{\bar{K}} = \bar{u}\bar{u} - \bar{v}\bar{v}$$

$$\bar{\bar{L}} = \bar{u}\bar{v} + \bar{v}\bar{u}$$

$$\bar{\bar{Z}}_s = Z_I\bar{\bar{I}}_t + Z_J\bar{\bar{J}} + Z_K\bar{\bar{K}} + Z_L\bar{\bar{L}}$$

EIGENVECTOR of $\bar{\bar{L}}$?

$$\bar{\bar{L}}\cdot(\bar{u}+\bar{v}) = \bar{v}+\bar{u}$$

$$\bar{\bar{L}}\cdot(\bar{u}-\bar{v}) = \bar{v}-\bar{u} = -(\bar{u}-\bar{v})$$

RECIPROCITY: $\bar{\bar{Z}}_s^T = \bar{\bar{Z}}_s$

LOSSLESS: $\bar{\bar{Z}}_s^* = -\bar{\bar{Z}}^T$

$\bar{\bar{Z}}_s = Z_J\bar{\bar{J}}$ non-reciprocal!

$$\bar{\bar{Z}}_s^* = \left(Z_I^*\bar{\bar{I}}_t + Z_J^*\bar{\bar{J}} + Z_K^*\bar{\bar{K}} + Z_L^*\bar{\bar{L}}\right)$$

$$= -Z_I\bar{\bar{I}}_t + Z_J\bar{\bar{J}} - Z_K\bar{\bar{K}} - Z_L\bar{\bar{L}}$$

$Z_I = -Z_I^*$ $\qquad$ $Z_K = -Z_K^*$ $\qquad$ $Z_L = -Z_L^*$

$Z_J = Z_J^*$ $\qquad$ Z_I, Z_K, Z_L purely imaginary

(Z_J real) $\qquad$ LOSSLESS

PEMC

$$\bar{D} = M\bar{B}$$
$$\bar{H} = -M\bar{E} \quad \Rightarrow \quad \bar{E} = -\frac{1}{M}\bar{H}$$

$M = 0$ PMC

$M^{-1} = 0$ PEC

$$\begin{pmatrix} \bar{D} \\ \bar{B} \end{pmatrix} = \lim_{q\to\infty} q \begin{pmatrix} M & 1 \\ 1 & 1/M \end{pmatrix} \begin{pmatrix} \bar{E} \\ \bar{H} \end{pmatrix}$$

$$\bar{D} = qM\bar{E} + q\bar{H} = q(\bar{H} + M\bar{E})$$
$$M\bar{B} = Mq\bar{E} + q\frac{M}{M}\bar{H} = \bar{D}$$

$$\bar{H} = -M\bar{E}$$

PEMC B.C.

$$\bar{n} \times (\bar{H} + M\bar{E}) = 0$$

$\bar{H}_t + M\bar{E}_t = 0$

$\varepsilon_0 \ \mu_0$

PEMC

$$\bar{E}_t = \bar{\bar{Z}}_s \cdot \bar{n} \times \bar{H}_t$$

$$Z_J \bar{\bar{J}} = Z_J \, \bar{n} \times \bar{\bar{I}}$$

$$\Rightarrow \bar{E}_t = Z_J \, \bar{n} \times (\bar{n} \times \bar{H}_t)$$
$$= -Z_J \bar{H}_t$$

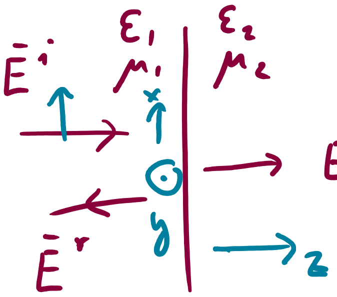
$$-\bar{n} \times \left[\bar{n} \times (\bar{E} + Z_J \bar{H})\right] = 0$$

REFLECTION

$$\bar E(\bar r) = \bar E_0 e^{-j\bar k\cdot\bar r}$$

ISOTROPIC MEDIUM

$$\bar k\cdot\bar k = \omega^2\mu\varepsilon$$



$$\bar E^T = T\bar E^i$$

$$= R\bar E^i$$

$$1+R=T$$

$$\bar E^i + \bar E^r = \bar E^T$$

$$E^i\bar u_x + E^r\bar u_x = \bar u_x \bar E^T$$

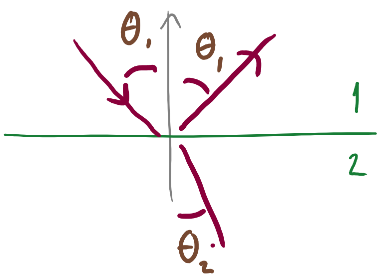
$$\nabla\times\bar E^i = -j\overbrace{\omega\mu}^{k\eta}\bar H^i$$

$$-j\bar k^i\times\bar E^i \quad \Rightarrow k\eta\bar H^i = \bar k^i\times\bar E^i \qquad \bar H^i = \frac{E^i}{\eta_1}\bar u_y$$

$$\frac{E^i}{\eta_1} - \frac{E^r}{\eta_1} = \frac{E^T}{\eta_2} = \frac{E^i+E^r}{\eta_2}$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} \qquad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$$

$$R = \frac{E^r}{E^i} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} \qquad T = 1+R = \frac{2\eta_2}{\eta_2+\eta_1}$$



$$k_1^2 = \omega^2\mu_1\varepsilon_1$$

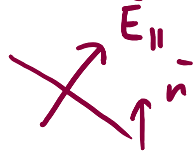
$$k_2^2 = \omega^2\mu_2\varepsilon_2$$

$$\sqrt{\mu_1\varepsilon_1}\sin\theta_1 = \sqrt{\mu_2\varepsilon_2}\sin\theta_2$$

(Snell)

$$R = \frac{\eta_{2t}-\eta_{1t}}{\eta_{2t}+\eta_{1t}}$$

PARALLEL POL.

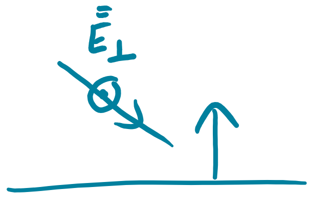


R (TM)

PARALLEL POL.

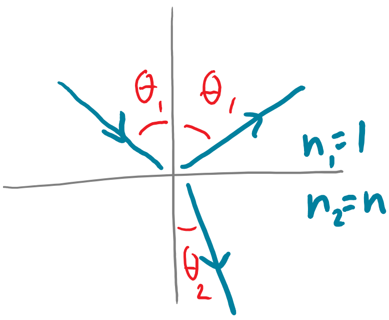
P (TM)

PERPENDICULAR POL.



S (TE)

$$\eta_{i_t} = \begin{cases} \eta_i \cos\theta_i & \text{PARALLEL} \\ \eta_i / \cos\theta_i & \text{PERPENDICULAR} \end{cases}$$



$$\eta_2 = \frac{1}{n}\eta_1$$

$$R_{\parallel} = \frac{\eta_2 \cos\theta_2 - \eta_1 \cos\theta_1}{\eta_2 \cos\theta_2 + \eta_1 \cos\theta_1} = \frac{\cos\theta_2 - n\cos\theta_1}{\cos\theta_2 + n\cos\theta_1}$$

Brewster: $\cos\theta_2 = n\cos\theta_1$

(Snell) $\longrightarrow$ $n\sin\theta_2 = \sin\theta_1$

$$n\sin\theta_1\cos\theta_1 = n\sin\theta_2\cos\theta_2$$

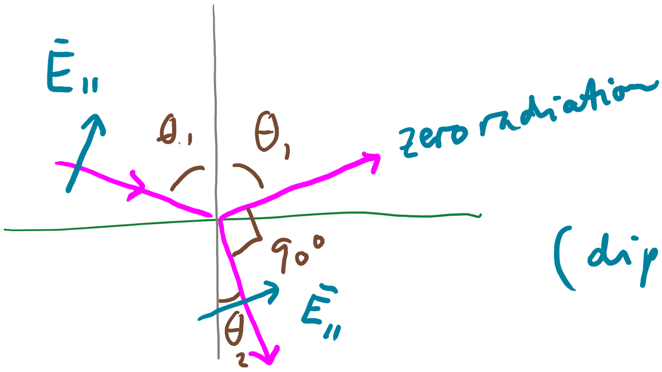
$$2\sin\theta_1\cos\theta_1 = 2\sin\theta_2\cos\theta_2$$

$$\sin 2\theta_1 = \sin 2\theta_2$$

$$2\theta_1 = \pi - 2\theta_2 \Rightarrow \theta_1 + \theta_2 = \frac{\pi}{2}$$

$$2\theta_1 = \pi - 2\theta_2 \Rightarrow \theta_1 + \theta_2 = \frac{\pi}{2}$$

$$\sin\theta_1 = \cos\theta_2$$
$$= n\cos\theta_1 \Rightarrow \tan\theta_1 = n$$



(dipole: no radiation into axis direction)