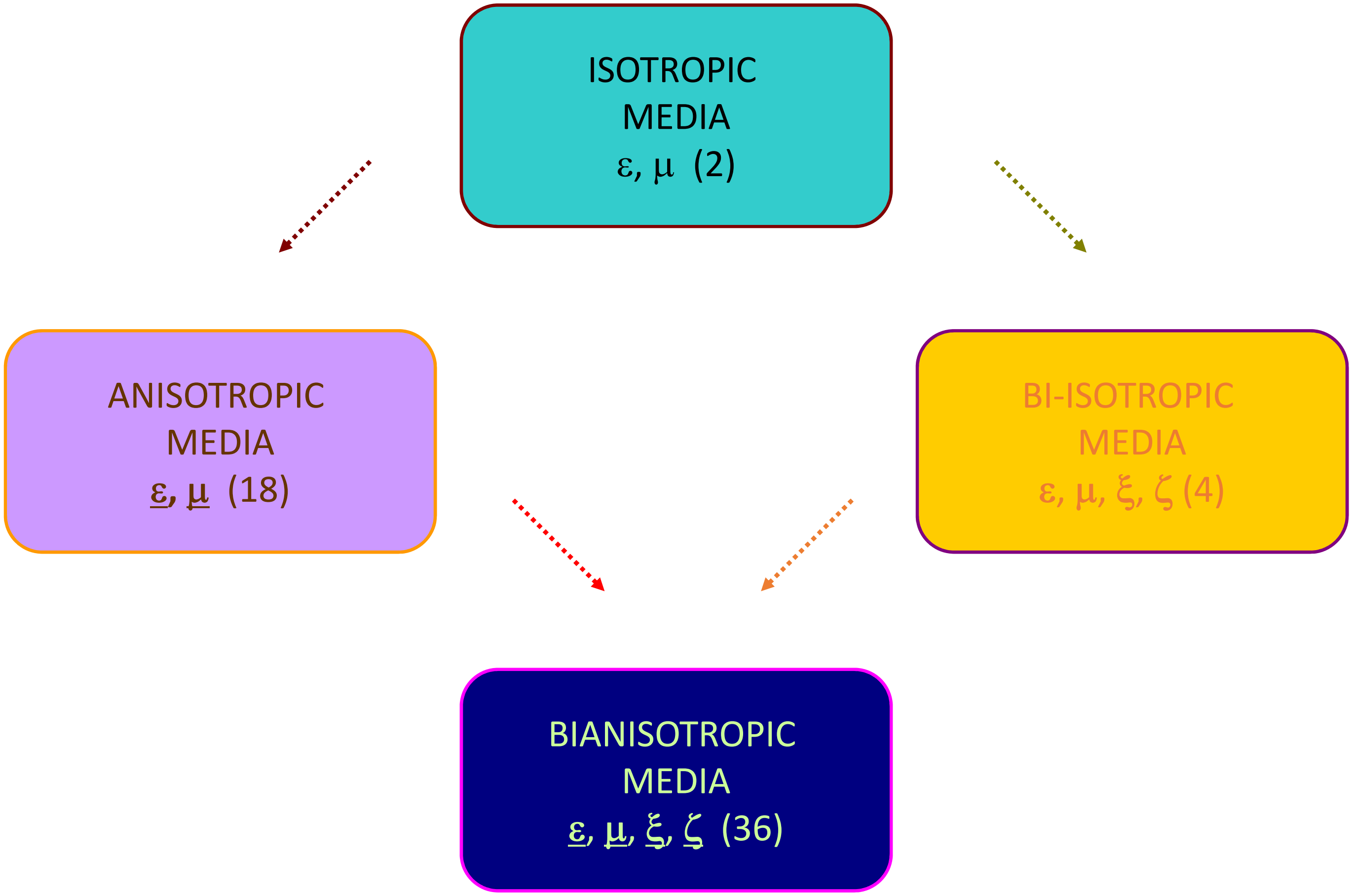
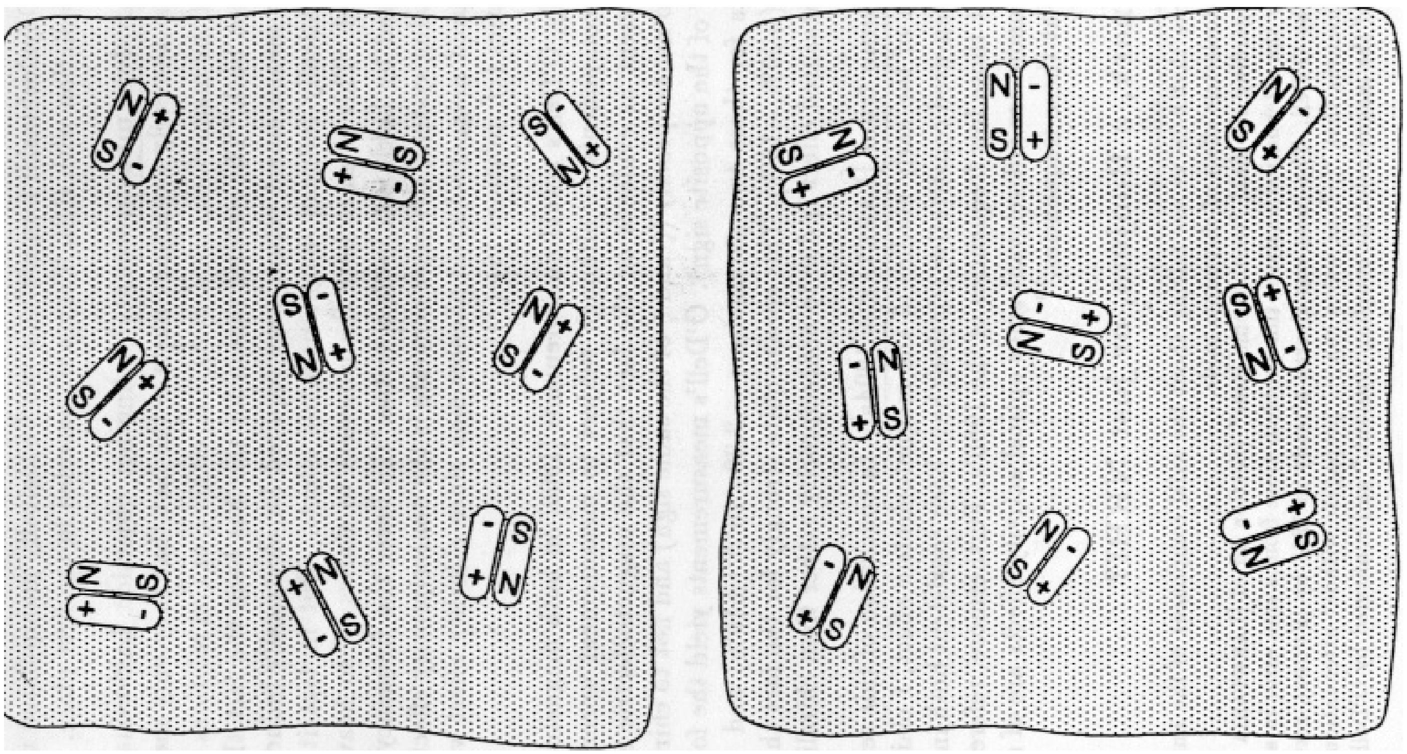

ISOTROPIC MEDIA
ε, μ (2)
ANISOTROPIC MEDIA
ε, μ (18)
BI-ISOTROPIC MEDIA
ε, μ, ξ, ζ (4)
BIANISOTROPIC MEDIA
ε, μ, ξ, ζ (36)

Tellegen (non-reciprocal) media

$$\begin{pmatrix} D \\ B \end{pmatrix} = \begin{pmatrix} \varepsilon & \chi \\ \chi & \mu \end{pmatrix} \begin{pmatrix} E \\ H \end{pmatrix}$$

χ non-reciprocity parameter (Tellegen parameter)

Tellegen (NRBI) material



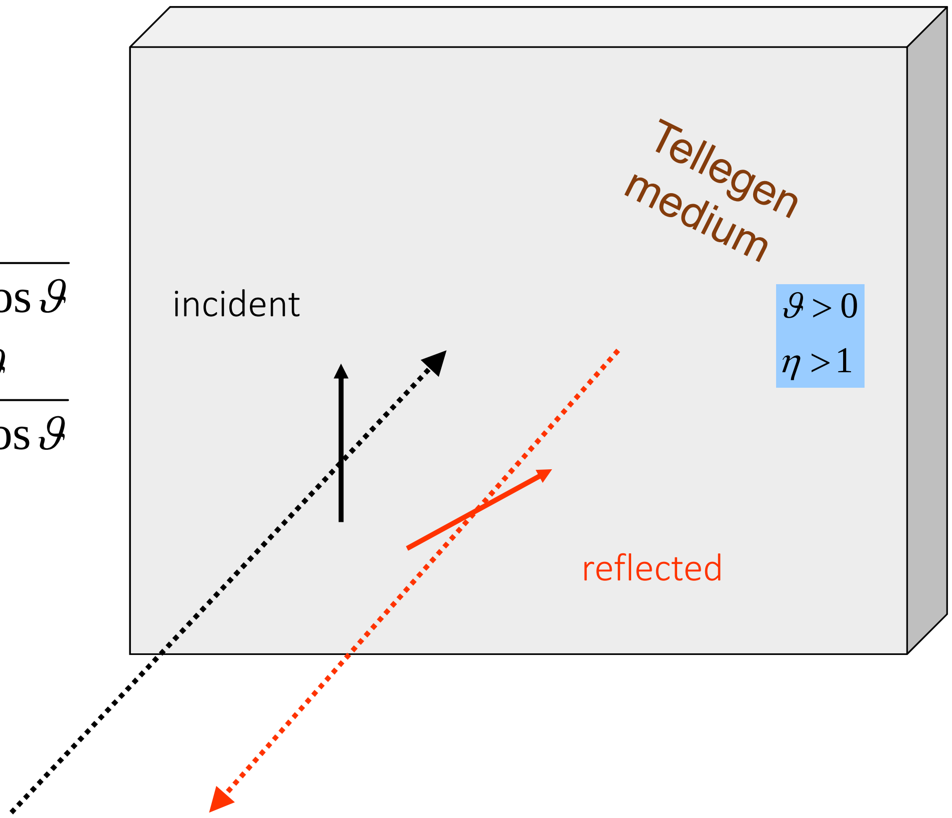
Tellegen: non-reciprocal reflection

$$R_{xx} = \frac{\eta^2 - 1}{\eta^2 + 1 + 2\eta\cos\vartheta}$$

$$R_{xy} = \frac{-2\eta\sin\vartheta}{\eta^2 + 1 + 2\eta\cos\vartheta}$$

$$\sin\vartheta = \frac{\chi}{\sqrt{\varepsilon\mu}}$$

$$\eta = \sqrt{\mu/\varepsilon}$$



$\vartheta > 0$

$\eta > 1$

Bi-isotropic media

$$\begin{pmatrix} D \\ B \end{pmatrix} = \begin{pmatrix} \varepsilon & \chi - \mathrm{j}\kappa \\ \chi + \mathrm{j}\kappa & \mu \end{pmatrix} \begin{pmatrix} E \\ H \end{pmatrix}$$

$\xi = \chi - \mathrm{j}\kappa$

$\zeta = \chi + \mathrm{j}\kappa$

κ chirality parameter (Pasteur)

χ non-reciprocity parameter (Tellegen)

Bianisotropic constitutive relations

$$D = \varepsilon \cdot E + \xi \cdot H$$

$$B = \zeta \cdot E + \mu \cdot H$$

$$\xi = \chi^{\mathrm{T}} - \mathrm{j}\kappa^{\mathrm{T}}$$

$$\zeta = \chi + \mathrm{j}\kappa$$

chirality dyadic

nonreciprocity dyadic

Lossless: $\xi = \zeta^{*\mathrm{T}} \Rightarrow \chi^{\mathrm{T}} - \mathrm{j}\kappa^{\mathrm{T}} = (\chi + \mathrm{j}\kappa)^{*\mathrm{T}} \Rightarrow \chi, \kappa$ real

Reciprocal: $\xi = -\zeta^{\mathrm{T}} \Rightarrow \chi^{\mathrm{T}} - \mathrm{j}\kappa^{\mathrm{T}} = -(\chi + \mathrm{j}\kappa)^{\mathrm{T}} \Rightarrow \chi = 0,\ \kappa$ arbitary

Classification of bi-anisotropic materials

	ε	μ	κ	χ
Symmetric part: 6 parameters	(RECIPROCAL) **Dielectric crystal**	**Magnetic medium**	**Chiral medium**	**Cr_2O_3**
Anti-symmetric part 3 parameters	(NON-RECIPROCAL) **Magneto-plasma**	**Biased ferrite**	**Omega medium**	**Moving medium**

A. Sihvola, I.V. Lindell (2008), Perfect electromagnetic conductor medium, *Ann. der Physik, **17**(9-10), 787-802*

Tellegen medium

- connection to
  - PEMC
  - axion
  - topological insulators

J. of Electromagn. Waves and Appl., Vol. 19, No. 7, 861–869, 2005

PERFECT ELECTROMAGNETIC CONDUCTOR

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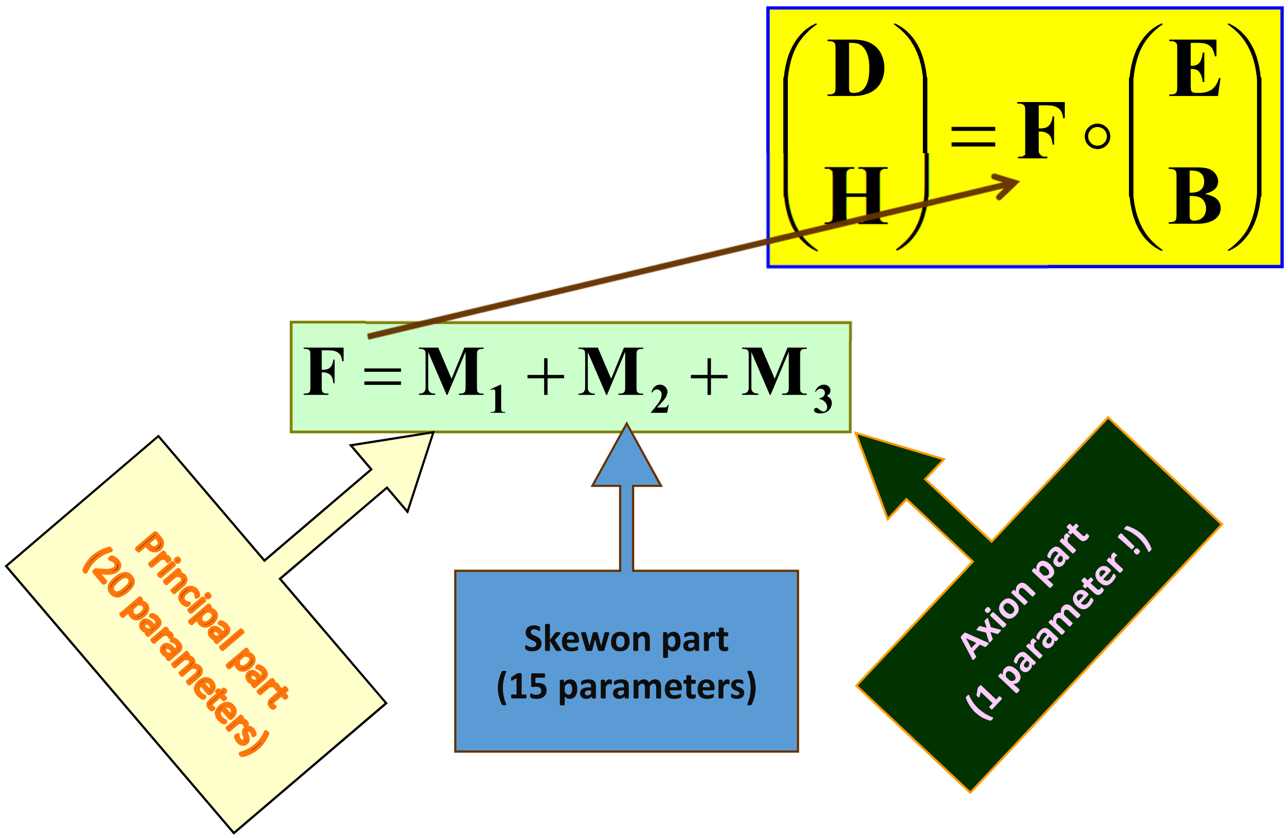
Abstract—In differential-form representation, the Maxwell equations are represented by simple differential relations between the electromagnetic two-forms and source three-forms while the electromagnetic medium is defined through a constitutive relation between the two-forms. The simplest of such relations expresses the electromagnetic two-forms as scalar multiples of one another. Because of its strange



F. W. Hehl & Yu. N. Obukhov (2003), *Foundations of Classical Electrodynamics,* Birkhäuser

Maxwell equations with differential forms

$$\mathbf{\Phi} = \mathbf{B} + \mathbf{E} \wedge \mathbf{d}\tau$$

$$\mathbf{\Psi} = \mathbf{D} - \mathbf{H} \wedge \mathbf{d}\tau$$

$$\mathbf{d} \wedge \mathbf{\Phi} = 0$$

$$\mathbf{d} \wedge \mathbf{\Psi} = \boldsymbol{\gamma}$$

Constitutive relation:

$$\Psi = \mathbf{M}\,\Phi$$

AXION-only: PEMC material

$$\Psi = \mathbf{M}\,\Phi \quad \longrightarrow \quad \Psi = M\,\Phi$$

$$\mathbf{D} = M\,\mathbf{B}, \quad \mathbf{H} = -M\,\mathbf{E}$$

$$\mathbf{\Phi} = \mathbf{B} + \mathbf{E} \wedge \mathbf{d}\tau$$

$$\mathbf{\Psi} = \mathbf{D} - \mathbf{H} \wedge \mathbf{d}\tau$$

PEMC material
(Perfect ElectroMagnetic Conductor)

PEC:

$$\begin{cases} E = 0 \\ B = 0 \end{cases} \rightarrow \begin{cases} \varepsilon = \infty \\ \mu = 0 \end{cases}$$

PMC:

$$\begin{cases} H = 0 \\ D = 0 \end{cases} \rightarrow \begin{cases} \mu = \infty \\ \varepsilon = 0 \end{cases}$$

$$H + \mathrm{M}E = 0 \quad \Rightarrow \quad D - \mathrm{M}B = 0$$

I.V. Lindell, A. H. Sihvola: Perfect electromagnetic conductor. *Journal of Electromagn. Waves Applicat.* 19(7), 861-869, 2005. A. Sihvola, I.V. Lindell: Perfect electromagnetic conductor medium, *Annalen der Physik (Berlin),* 17, 787-802, 2008.

Magnetoelectric relations:

$$\begin{pmatrix} \mathbf{D} \\ \mathbf{B} \end{pmatrix} = \begin{pmatrix} \varepsilon & \xi \\ \zeta & \mu \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = q \begin{pmatrix} M & 1 \\ 1 & 1/M \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}$$

$$q \to \infty$$

$$\mathrm{D} = q(M\,\mathrm{E} + \mathrm{H}) \quad \& \quad \mathrm{B} = q\left(\mathrm{E} + \frac{1}{M}\mathrm{H}\right)$$

$$\Rightarrow \; \mathrm{D} = M\,\mathrm{B} \;\; \& \;\; \mathrm{H} = -M\,\mathrm{E}$$

Practical Realization of Perfect Electromagnetic

C ... es

using I ... ciprocal

Meta ... phene

C. Caloz [#1], A. ... N. Chamanara [#1]

[#] Poly-Grames R ... , H3T 1J4, Canada

[*] Department ... 558611, Japan

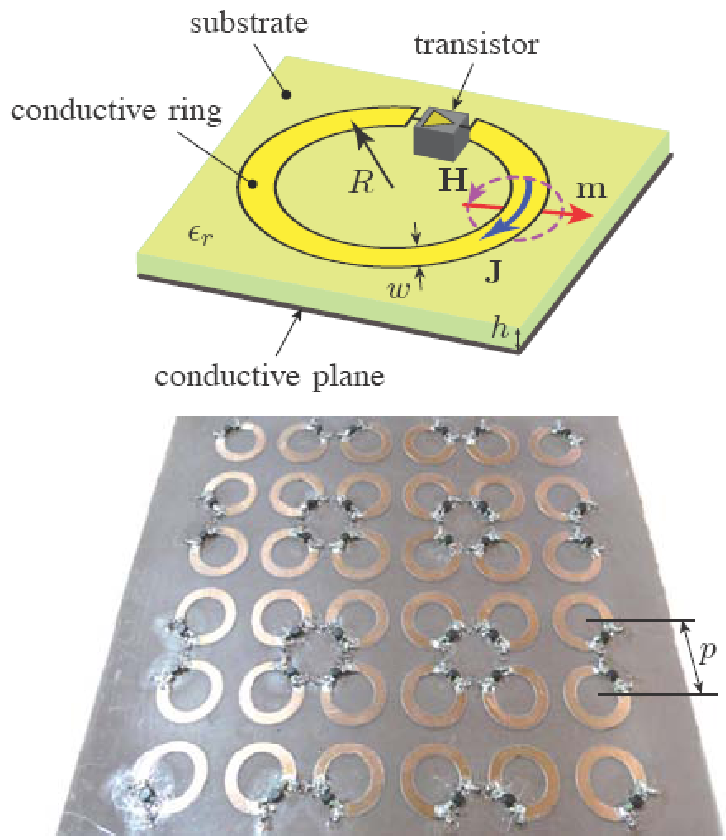


Fig. 4. Magnet-less non-reciprocal metamaterial (MNM) PEMC. Unit cell (top) and experimental prototype (bottom).