Int. J. Elect. Enging. Educ., Vol. 32, pp. 39–42. Manchester U.P., 1995. Printed in Great Britain

AN ELEMENTARY APPROACH IN TEACHING BREWSTER'S ANGLE

ARI SIHVOLA
Electromagnetics Laboratory, Helsinki University of Technology, Finland

Wave reflection and refraction from planar interfaces are important physical phenomena, and the quantitative discussion of this topic is present in electromagnetics and field theory courses in electrical engineering schools. To some extent, these phenomena are also taught in basic physics courses at the high school and college levels. Both for radio waves and optical rays, reflection and transmission problems are encountered in nearly all applications, for example in everyday light phenomena. Therefore a thorough understanding of this canonical case of planar interface effects would be helpful to students.

The analysis of the reflection problem leads to ideas like Snell's law, Fresnel reflection coefficients, total internal reflection, and Brewster's angle. The first three of these associated concepts can be fairly easily derived from simple geometrical and basic physical principles. However, although the idea of Brewster's angle is intuitively rather clear, the explicit formula for this very angle is not that readily available. In textbook treatments, the derivation requires unpleasant algebraic manipulation of reflection coefficient expressions loaded with trigonometric functions of angles.

The purpose of this note is to present a simple way of teaching the expression for Brewster's angle, or polarizing angle as it can be called due to the fact that unpolarized wave incident in this angle is reflected in a totally polarized state. There is no need to remember or derive complicated expressions; basic rules are sufficient.

Consider the geometry of Fig. 1, where the plane wave is incident on a plane boundary between two media with permittivities ε_1, ε_2 and permeabilities μ_1, μ_2. The reflection coefficient (the amplitude ratio of the electric field of the reflected wave to that of the incident wave) is an easily remembered formula from transmission line theory:

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1} \qquad (1)$$

where Z_1 is the impedance of the medium from where the wave is coming from, and Z_2 is the impedance of the medium ahead.

In plane wave propagation problems, this formula also applies. The impedance is the wave impedance of the material $\eta = \sqrt{\mu/\varepsilon}$. However, as now

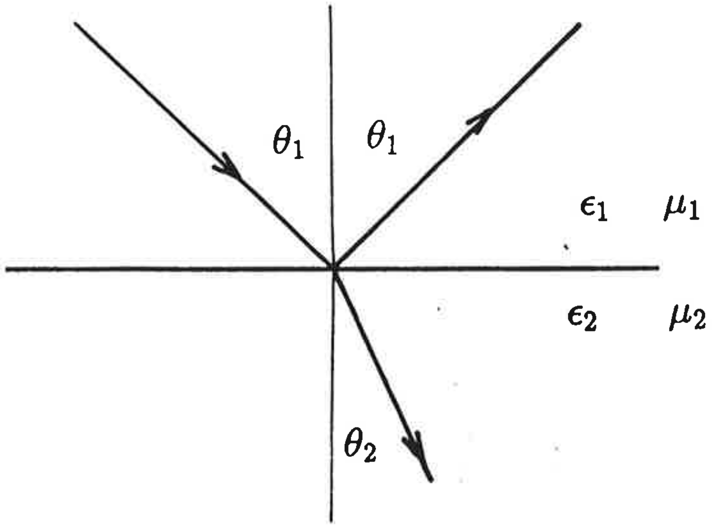

FIG. 1 The geometry of the problem: a plane wave incident on a planar interface between two media.

the wave is obliquely incident, the transverse impedance has to be used:

$$Z_i = \sqrt{\frac{\mu_i}{\varepsilon_i}} \cos\theta_i \quad \text{(parallel polarization)}, \tag{2}$$

$$Z_i = \sqrt{\frac{\mu_i}{\varepsilon_i}} \frac{1}{\cos\theta_i} \quad \text{(perpendicular polarization)}. \tag{3}$$

Here, 'parallel' and 'perpendicular' refer to the electric field vector relation to the plane of incidence. θ_1 is the angle of incidence and θ_2 is the angle of refraction. This is an easy way of memorizing the Fresnel reflection coefficients of a dielectric and/or magnetic interface. Normally, these expressions are expressed in more complicated forms (see Refs.).

Brewster's angle is the incidence angle with which there is no reflected wave. It is well known that for a dielectric interface ($\mu_1 = \mu_2$), there is a Brewster angle only for parallel polarization, and for a magnetic interface ($\varepsilon_1 = \varepsilon_2$), only for perpendicular polarization. Let us only consider a dielectric interface, which is the most common case in actual practice. Hence, there is only one material parameter in the problem, the refractive index $n = \sqrt{\varepsilon_2/\varepsilon_1}$.

Brewster's angle predicts zero reflection coefficient for parallel polarization:

$$R_{\text{par}} = 0 \Rightarrow \eta_2 \cos\theta_2 = \eta_1 \cos\theta_1 \tag{4}$$

or

$$n \cos\theta_1 = \cos\theta_2. \tag{5}$$

The other equation that is needed is Snell's law:

$$\sqrt{\mu_1 \varepsilon_1} \sin\theta_1 = \sqrt{\mu_2 \varepsilon_2} \sin\theta_2 \tag{6}$$

or

$$\sin \theta_1 = n \sin \theta_2. \tag{7}$$

Multiplying (5) by (7) gives

$$\sin \theta_1 \cos \theta_1 = \sin \theta_2 \cos \theta_2 \tag{8}$$

or

$$\sin 2\theta_1 = \sin 2\theta_2. \tag{9}$$

This leads to either $\theta_1 = \theta_2$ (no interface), or to the Brewster's angle condition: $2\theta_1 = \pi - 2\theta_2$. This fact, that θ_1 and θ_2 form a right angle, has a well-known geometric-physical interpretation (Fig. 2).

However, the expression for Brewster's angle emerges directly from dividing equations (7) and (5):

$$\tan \theta_1 = n^2 \tan \theta_2, \tag{10}$$

from which, using the fact $\theta_2 = \pi/2 - \theta_1$, or $\tan \theta_2 = \cot \theta_1 = 1/\tan \theta_1$, we are left with

$$\tan \theta_1 = n, \tag{11}$$

which is the well-known equation for Brewster's angle (see Refs.).

In summary, it has been my experience as an electromagnetics teacher that, by playing with these two simple and easily memorized equations (5) and (7),

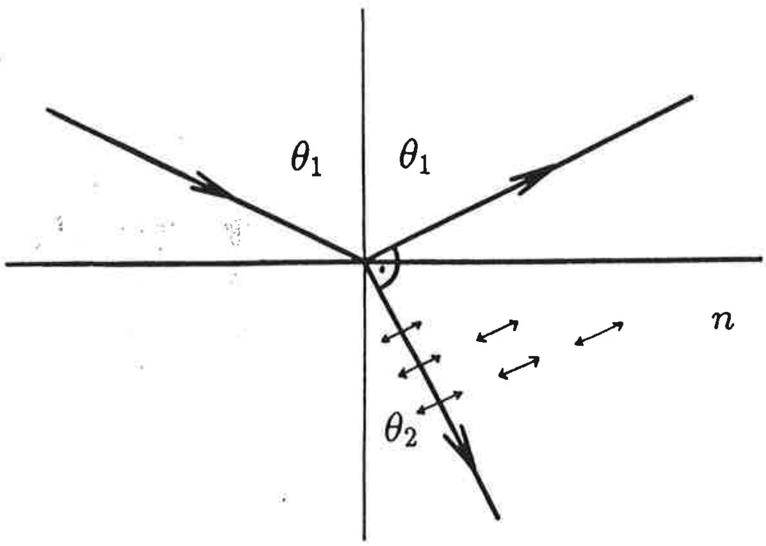

FIG. 2 Physical interpretation of the Brewster angle: as the molecules of the lower medium are polarized by the refracted electric field such that they point exactly to the reflected wave direction, then there will be no reflected wave. This is because the dipole radiation pattern has a null (no radiation) in the axis direction.

it is easier for the students to master quantitatively and interpret physically light reflection and transmission problems encountered in everyday life.

REFERENCES

Jackson, J. D., *Classical Electrodynamics, Second Ed.*, John Wiley, pp. 281–282 (1975)

Cheng, D. K., *Field and Wave Electromagnetics*, Addison-Wesley, pp. 413–415 (1989)

Iskander, M. F., *Electromagnetic Fields and Waves*, Prentice-Hall, pp. 455–456 (1992)

Kraus, J. D., *Electromagnetics, 4th Ed.*, McGraw-Hill, pp. 613–616 (1992)

ABSTRACTS – ENGLISH, FRENCH, GERMAN, SPANISH

An elementary approach in teaching Brewster angle

Brewster angle is an important concept in learning optics and electromagnetic wave propagation phenomena. It conveys the emphasis on polarisation of the fields. An intuitive way is presented to derive Brewster angle by connecting impedances and reflection coefficients. The radical polarisation differences emerge clearly from this approach.

Une approche élémentaire dans l'enseignement de l'angle de Brewster

L'angle de Brewster est un concept important dans l'enseignement de l'optique et de la propagation des ondes électromagnétiques. Il conduit à l'emphase sur la polarisation des champs. Une approche intuitive est présentée ici pour déduire l'angle de Brewster par connexion d'impédances et coefficients de réflexion. Les différences de polarisation apparaissent clairement par cette approche.

Ein elementares Vorgehen beim Lehren des Brewster-Winkels

Brewster-Winkel ist ein wichtiges Konzept beim Lernen optischer und elektromagnetischer Wellenausbreitungsphänomene. Es übermittelt die Hervorhebung der Polarisierung der Felder. Ein intuitiver Weg wird vorgestellt, um den Brewster-Winkel durch Verbindung von Impedanzen und Reflexionsfaktoren zu ermitteln. Die Wurzelpolarisationsunterschiede kommen bei diesem Vorgehen deutlich zum Vorschein.

Una aproximación sencilla para la enseñanza del ángulo de Brewster

El ángulo de Brewster es un concepto importante para el aprendizaje de los fenómenos de propagación de ondas ópticas y magnéticas. Enfatiza en la polarización de los campos. Se presenta un modo intuitivo para derivar el ángulo de Brewster conectando impedancias y coeficientes de reflexión. Las diferencias radicales de polarización aparecen claramente con esta aproximación.