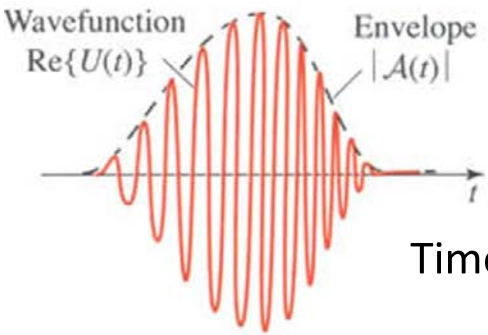
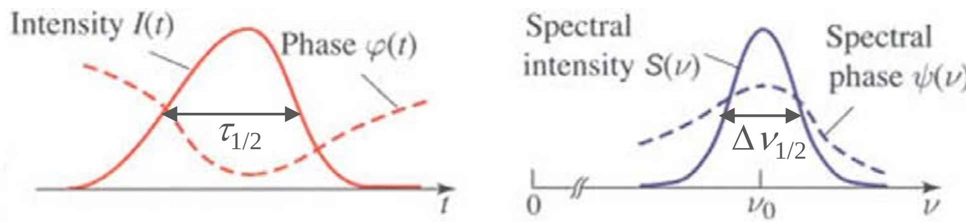
Chapter 22

ULTRAFAST OPTICS I

Pulse characteristics





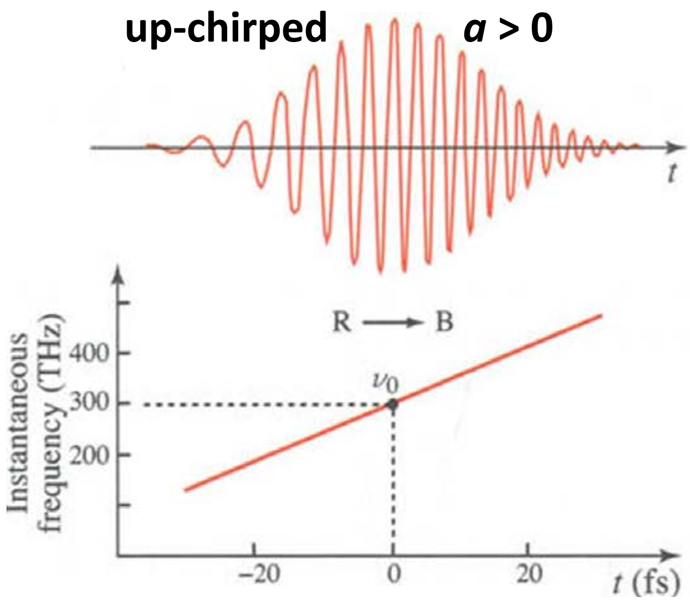
Time-bandwidth product $\tau_{1/2}\Delta\nu_{1/2} \geq 0.44$

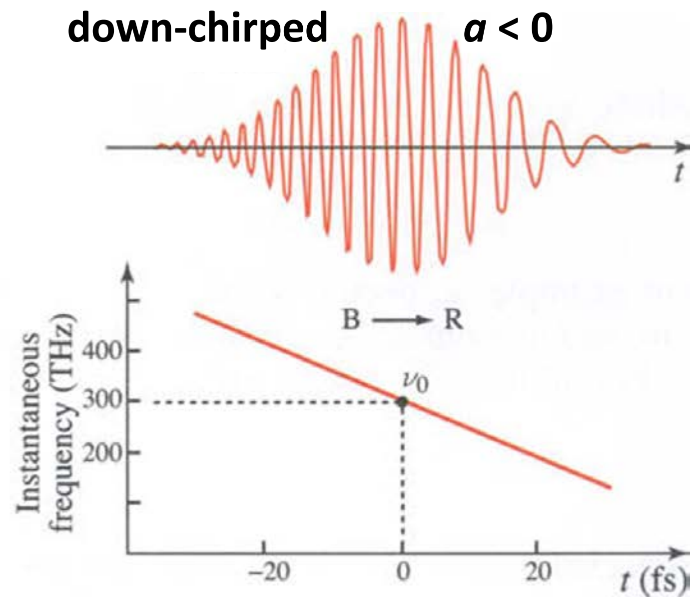
$=$ for a Gaussian FT-limited pulse

The total phase is $\phi(t) = \omega_0 t + \varphi(t)$, and the ***instantaneous frequency*** is $\mathrm{d}\phi(t)/\mathrm{d}t$:

$$\omega_i = \omega_0 + \frac{d\varphi}{dt} = \omega_0 + \varphi'(0) + \varphi''(0)t + \cdots$$

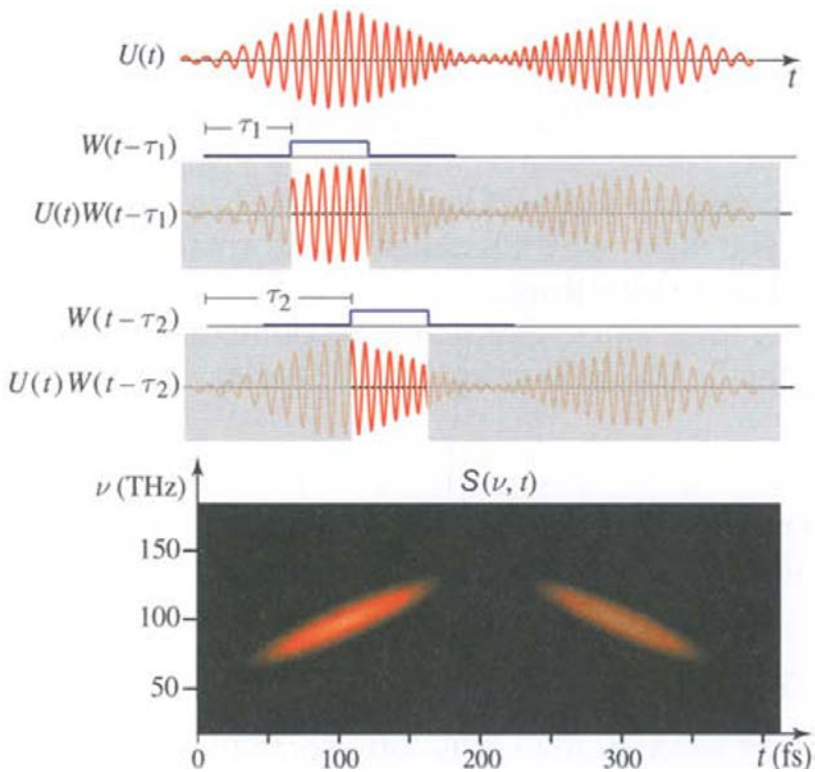
The ***lowest-order chirp parameter*** for a pulse is $a = \varphi'' \tau^2/2$.





Time-varying spectrum

|Short-time Fourier transform|^2 = Spectrogram

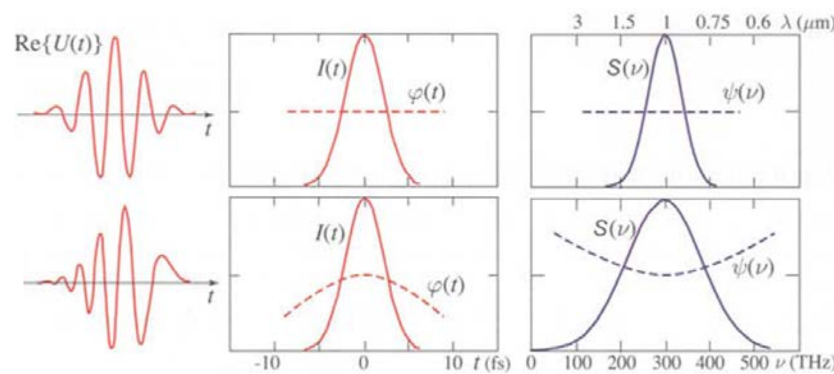


Short-time FT:

$$\Phi(\nu,\tau) = \int U(t)W(t-\tau)\exp(-j2\pi\nu t)dt.$$

Chirped Gaussian pulse

$A(t) = A_0 \exp[-(1-ja)t^2/\tau^2]$	Complex envelope
$I(t) = I_0 \exp(-2t^2/\tau^2)$	Intensity
$\int I(t)dt = \sqrt{\pi/2}I_0\tau$	Energy density
$\tau_{1/e} = \sqrt{2}\tau$	$1/e$ half width
$\tau_{\mathrm{FWHM}} = 1.18\tau$	FWHM width
$\varphi(t) = at^2/\tau^2$	Phase
$A(\nu) = \dfrac{A_0\tau}{2\sqrt{\pi(1-ja)}}\exp\left[-\dfrac{\pi^2\tau^2\nu^2}{1-ja}\right]$	Fourier transform
$S(\nu) = \dfrac{I_0\tau^2}{4\pi\sqrt{1+a^2}}\exp\left[-\dfrac{2\pi^2\tau^2(\nu-\nu_0)^2}{1+a^2}\right]$	Spectral intensity
$\Delta\nu_{1/e} = \dfrac{2}{\tau}\sqrt{1+a^2}$	$1/e$ half width
$\Delta\nu = \dfrac{0.375}{\tau}\sqrt{1+a^2} = \dfrac{0.44}{\tau_{\mathrm{FWHM}}}\sqrt{1+a^2}$	FWHM Spectral width
$\psi(\nu) = -2\pi^2\tau^2[a/(1+a^2)]\nu^2$	Spectral phase
$\nu_i = \nu_0 + (a/\pi\tau^2)t$	Instantaneous frequency



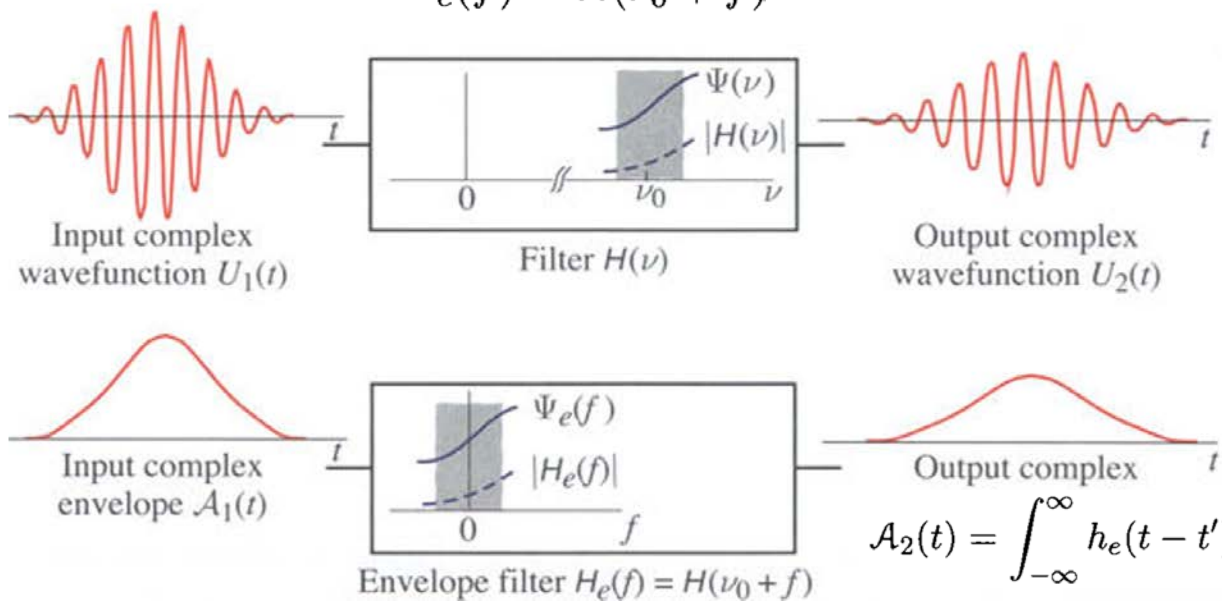
Pulse shaping and compression

When the frequency components of $U_1(t) = A_1(t)\exp(j2\pi\nu_0 t)$ are changed by a system,

$$V_2(\nu) = H(\nu)\,V_1(\nu).$$

If for a pulse, $f = \nu - \nu_0 << \nu_0$, we use $A_2(f) = H_e(f)A_1(f)$, with the transfer function

$$H_e(f) = H(\nu_0 + f)$$



$$A_2(t) = \int_{-\infty}^{\infty} h_e(t-t')\,A_1(t')\,dt'$$

An ideal *non-disturbing filter* introduces constant attenuation and time delay τ_d of each frequency component: $H_e(f) = H_0 \exp\left(-j2\pi f \tau_d\right)$.

A *Gaussian chirp filter* phase-shifts each frequency component *quadratically*:

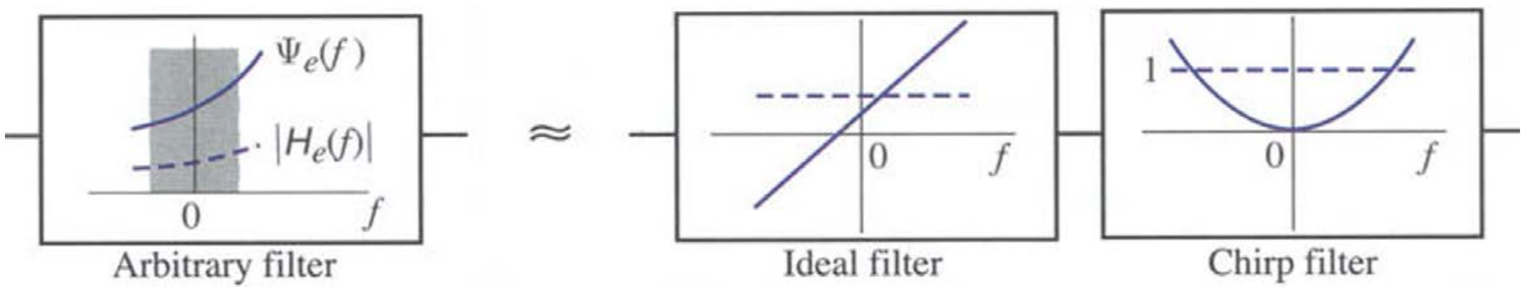
$$H_e(f) = \exp\left(-jb\pi^2 f^2\right) \quad \text{and} \quad h_e(t) = \frac{1}{\sqrt{j\pi b}}\exp(jt^2/b).$$

For an *arbitrary phase filter* with a slowly varying phase Ψ, one can use the Taylor series

$$H_e(f) \approx |H_0| \exp\left[-j(\Psi_0 + \Psi' f + \tfrac{1}{2}\Psi'' f^2)\right],$$

where $\Psi' = d\Psi/d\nu|_{\nu_0}$. Hence, the filter is equivalent to an ideal filter followed by a chirp filter with

$$\tau_d = \Psi'/2\pi \quad \text{and} \quad b = \frac{\Psi''}{2\pi^2}.$$



❖ **Gaussian chirp filtering of an unchirped Gaussian pulse**:

$$A_1(t) = A_{10}\exp(-t^2/\tau_1^2), \text{ and its FT is } A_1(f) = (A_{10}\tau_1/2\sqrt{\pi})\exp(-\pi^2\tau_1^2 f^2)$$

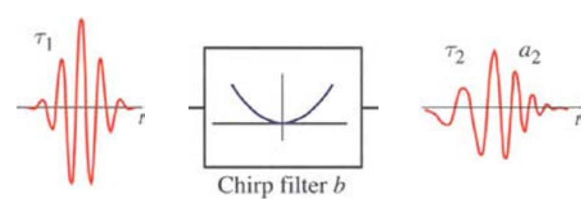
Filtered: $A_2(f) = A_{10}\dfrac{\tau_1}{2\sqrt{\pi}}\exp[-\pi^2(\tau_1^2 + jb)f^2]$

In general, a chirped Gaussian pulse has $A_2(f) = A_{20}\dfrac{\tau_2}{2\sqrt{\pi(1-ja_2)}}\exp\left(-\dfrac{\pi^2\tau_2^2 f^2}{1-ja_2}\right)$.

$$\Rightarrow \tau_1^2 + jb = \frac{\tau_2^2}{1-ja_2} \text{ and } A_{20} = \frac{A_{10}}{\sqrt{1+jb/\tau_1^2}},$$

$$\Rightarrow \tau_2 = \tau_1\sqrt{1+b^2/\tau_1^4} \text{ and } a_2 = b/\tau_1^2.$$

pulse width — chirp parameter



❖ **Gaussian chirp filtering of a chirped Gaussian pulse**:

$$A_1(t) = A_{10} \exp[-(1 - ja_1)t^2/\tau_1^2] \longrightarrow A_2(t) = A_{20} \exp[-(1 - ja_2)t^2/\tau_2^2],$$

where

$$\frac{\tau_2^2}{1 - ja_2} = \frac{\tau_1^2}{1 - ja_1} + jb.$$

$$\Rightarrow \tau_2 = \tau_1 \sqrt{1 + 2a_1 \frac{b}{\tau_1^2} + (1 + a_1^2)\frac{b^2}{\tau_1^4}} \text{ and } a_2 = a_1 + (1 + a_1^2)\frac{b}{\tau_1^2}$$

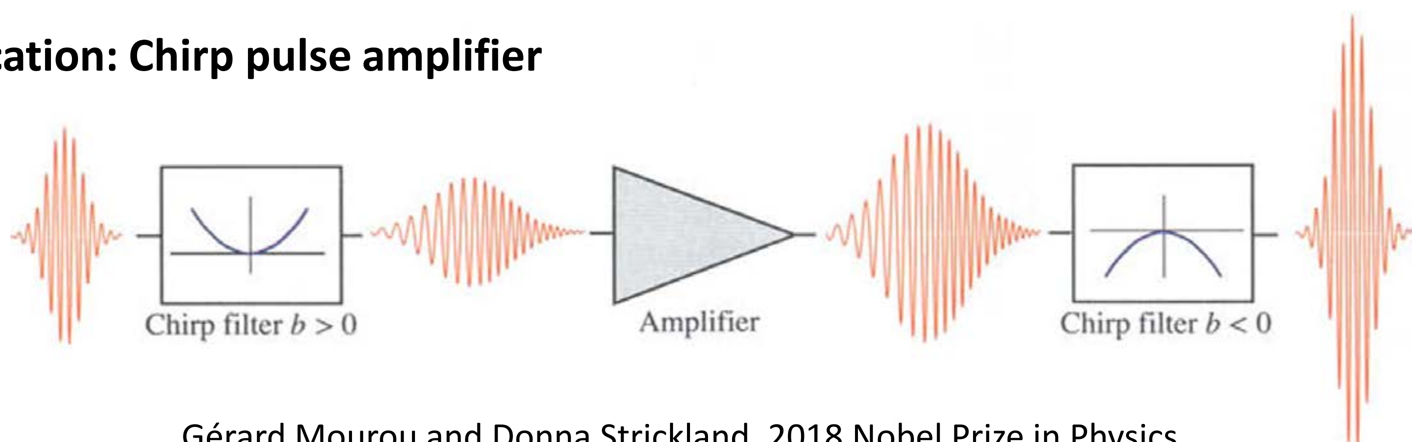
The pulse is unchirped and maximally compressed by the filter when $a_2 = 0$ so that

$$b_{\min} = -a_1 \tau_0^2 = -\frac{a_1}{1 + a_1^2}\tau_1^2$$

$$\Rightarrow \tau_0 = \frac{\tau_1}{\sqrt{1 + a_1^2}} = \tau_2$$

$$\Rightarrow \tau_2 = \tau_0 \sqrt{1 + (b - b_{\min})^2/\tau_0^4} \text{ and } a_2 = (b - b_{\min})/\tau_0^2.$$

Application: Chirp pulse amplifier

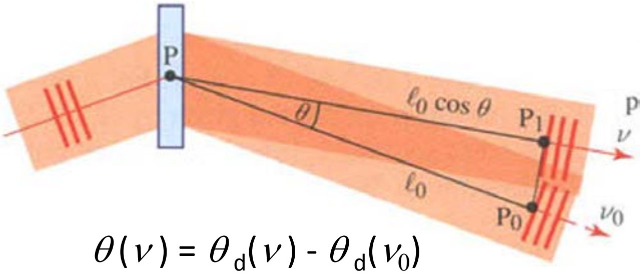


Gérard Mourou and Donna Strickland, 2018 Nobel Prize in Physics

Chirp filters

Angular-dispersion chirp filters:

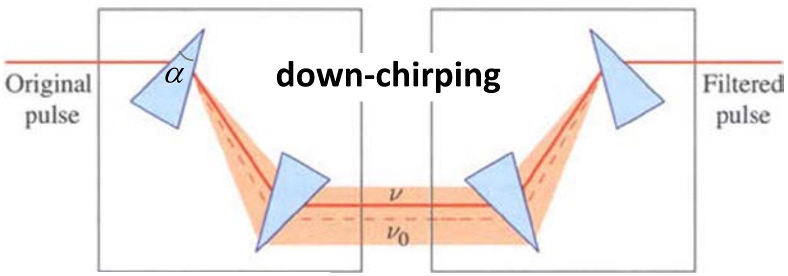
$$H(\nu) = \exp[-j\Psi(\nu)], \quad \Psi(\nu) = \frac{2\pi\nu}{c}\ell_0 \cos\theta(\nu)$$



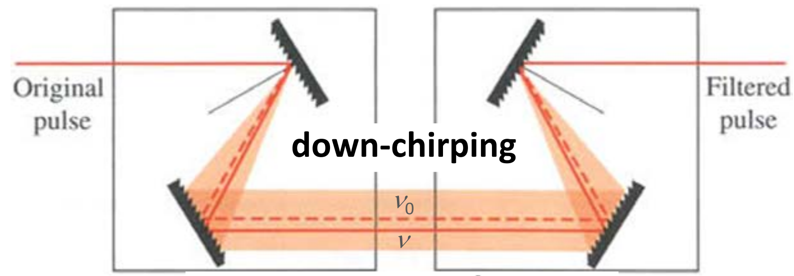
$\theta(\nu) = \theta_d(\nu) - \theta_d(\nu_0)$

$$\Psi'' \approx -\frac{2\pi\nu}{c}\ell_0 \left(\frac{d\theta}{d\nu}\right)^2$$

$$\Rightarrow \; b \approx -\frac{\ell_0}{\pi\lambda_o}\alpha_\nu^2 < 0, \text{ where } \alpha_\nu = d\theta/d\nu\,.$$

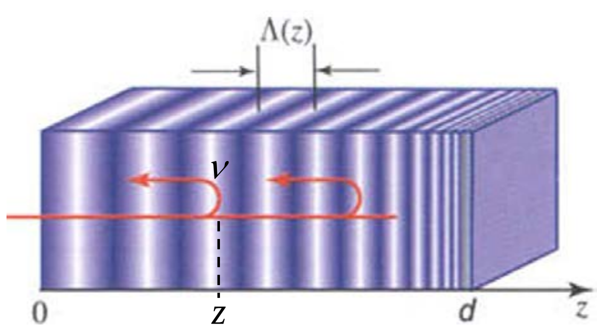


$$\alpha_\nu = \frac{n-N}{\nu_0}\alpha, \quad N = n - \lambda_o dn/d\lambda_o$$



$$\alpha_\nu = -\frac{1}{\nu_0}\frac{\lambda_o}{\sqrt{\Lambda^2 - (\lambda_o/2)^2}}$$

Bragg-grating chirp filters: Spatial frequency varies linearly, $\Lambda^{-1}(z) = \Lambda_o^{-1} + \xi z$.



Each frequency component travels a distance $2z$, and at z, it satisfies $\Lambda(z) = m\lambda/2 = mc/2\nu$

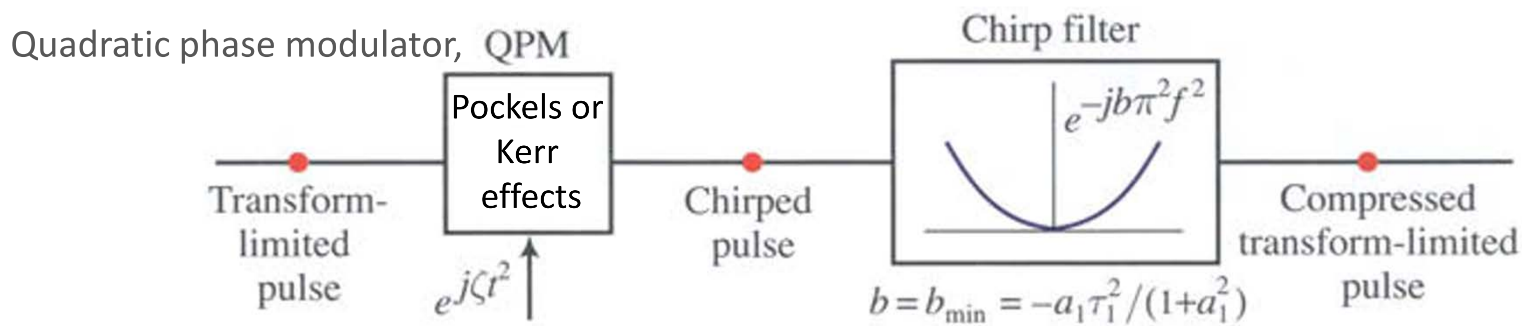
$$\Rightarrow \; z = 2\nu/(mc\xi) - 1/(\xi\Lambda_o) \text{ and}$$

$$\Psi = (2\pi\nu/c)(2z) = (8\pi/mc^2\xi)\nu^2 + (4\pi/c\xi\Lambda_o)\,\nu.$$

$$\Rightarrow \; b = \frac{8}{m\pi\xi c^2} > 0 \text{ for } \xi > 0$$ **(up-chirping)**

Pulse compression

Compression of a transform-limited pulse:



Pulse width:	τ_1	τ_1	$\tau_0 = \tau_1/\sqrt{1+a_1^2}$
Chirp parameter:	0	$a_1 = \zeta\tau_1^2$	0
Spectral width:	$\Delta\nu$	$\Delta\nu\sqrt{1+a_1^2}$	$\Delta\nu\sqrt{1+a_1^2}$

If the original pulse is chirped:

• A chirp filter can compress the pulse by unchirping it.

• In contrast, a quadratic phase filter cannot change the pulse duration, but it changes the chirp parameter as

$$a_2 = a_1 + \zeta\tau_1^2.$$

The filter will be able to unchirp the pulse, but it will also broaden the spectrum.

Optical fiber as a chirp filter

In the presence of *dispersion*, propagation multiplies each frequency component with a propagation phase factor $\exp[-j\beta(\nu)z]$, i.e.,

$$V(z,\nu) = \exp[-j\beta(\nu)z]\, V(0,\nu).$$

For narrowband light, $U(z,t) = A(z,t)\exp(-j\beta_0 z)\exp(j2\pi\nu_0 t)$.

$$\Rightarrow V(z,\nu) = A(z,\nu-\nu_0)\exp(-j\beta_0 z)$$

$$\Rightarrow A(z,f) = H_e(f)A(0,f)$$

$$H_e(f) = \exp\{-j[\beta(\nu_0+f)-\beta(\nu_0)]z\}.$$

For slowly varying $\beta(\nu)$, the Taylor expansion of $\Psi(f) = [\beta(\nu_0+f)-\beta(\nu_0)]z$ yields

$$H_e(f) \approx \exp[-j(\Psi' f + \tfrac{1}{2}\Psi'' f^2)] = \exp(-j2\pi\tau_d f)\exp(-jb\pi^2 f^2).$$

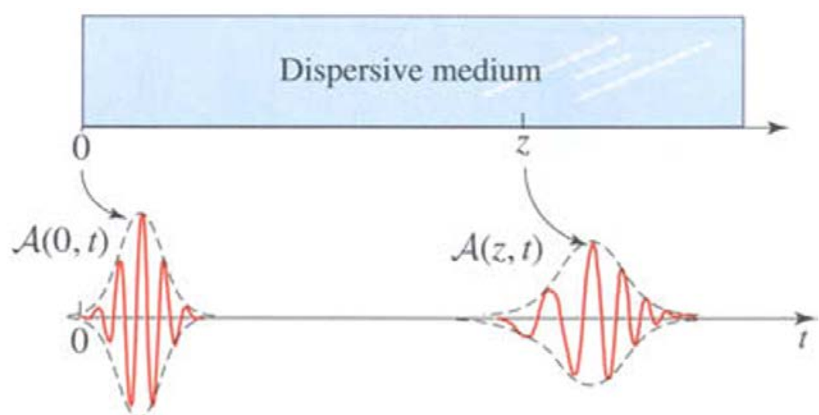
The *time delay* of the pulse is $\tau_d = z/v$, where the group velocity is $v = 1/\beta' = \frac{c_o}{N}$.

Propagation acts as a *chirp filter* with

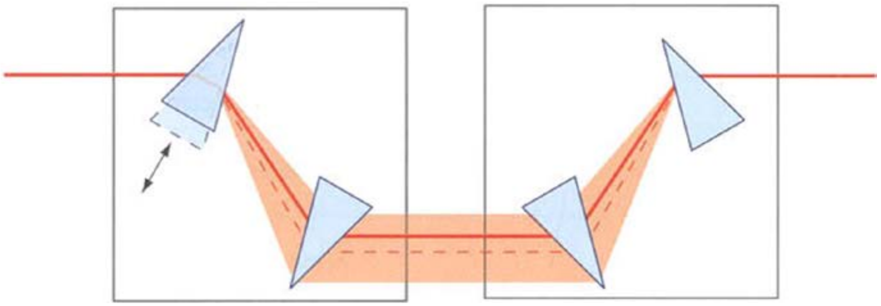
$$b = 2\beta'' z = \frac{D_\nu}{\pi} z,$$

where the **group velocity dispersion** (GVD) coefficient is

$$D_\nu = 2\pi\beta'' = \frac{d}{d\nu}\left(\frac{1}{v}\right) = \frac{\lambda_o^3}{c_o^2}\frac{d^2 n}{d\lambda_o^2}$$



Adjustable prism chirp filter (through GVD):



Angular-dispersion chirp,

$$b \approx -\frac{\ell_0}{\pi \lambda_o} \alpha_\nu^2, \quad \alpha_\nu = \frac{n - N}{\nu_0} \alpha,$$

and a material-dispersion chirp

$$b_m = 2\beta'' L = D_\nu L / \pi > 0.$$

Transform-limited Gaussian input pulse in an optical fiber (all x_0 are at $z = 0$)

$b = D_\nu z / \pi$

$$A_{20} = \frac{A_{10}}{\sqrt{1 + jb/\tau_1^2}}$$

$$\tau_2 = \tau_1 \sqrt{1 + b^2/\tau_1^4}$$

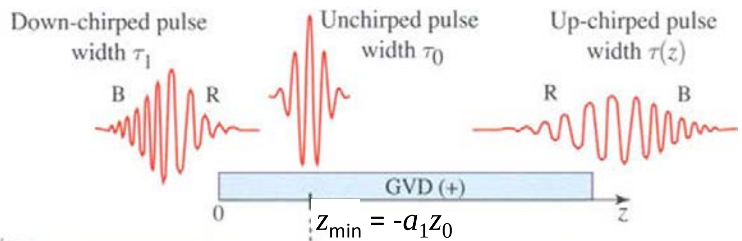
$a_2 = b/\tau_1^2$ (page 5)

Dispersion length z_0 is such that

$$\tau(z) = \tau_0 \sqrt{2}.$$

$A(z,t) = A_0 \sqrt{\frac{-jz_0}{z - jz}} \exp\left[j \frac{\pi}{D_\nu} \frac{(t - z/v)^2}{z - jz_0}\right]$	Complex envelope
$I(z,t) = I_0 \frac{\tau_0}{\tau(z)} \exp\left[-2 \frac{(t - z/v)^2}{\tau^2(z)}\right]$	Intensity
$\int I(t)dt = \sqrt{\pi/2} I_0 \tau_0$	Energy density
$\tau(z) = \tau_0 \sqrt{1 + (z/z_0)^2}$	Pulse width
$a(z) = z/z_0$	Chirp parameter
$z_0 = \pi \frac{\tau_0^2}{D_\nu} = \frac{\tau_0^2}{2\beta''}$	Dispersion length $\lvert z_0 \rvert$
$\Delta\nu = \frac{0.375}{\tau_0}$	Spectral width

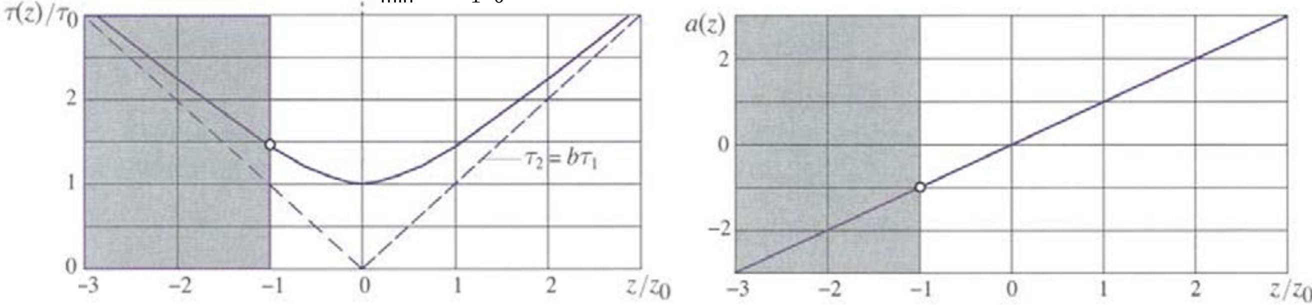
❖ Chirped Gaussian input pulse:



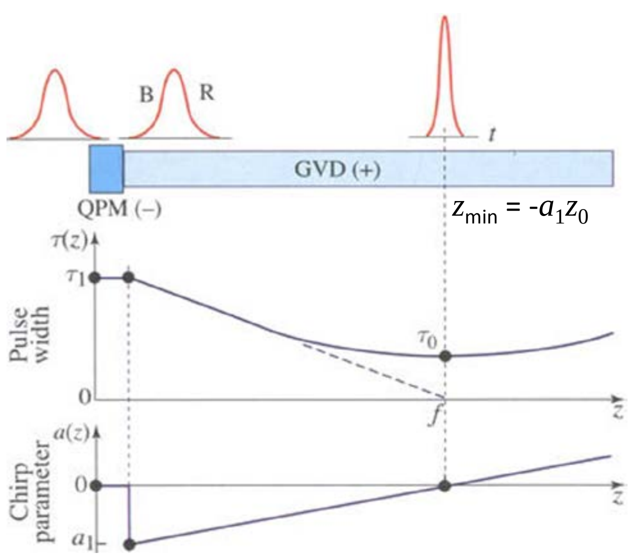
$$\tau(z) = \tau_0 \sqrt{1 + (z - z_{\min})^2 / z_0^2},$$

$$a(z) = (z - z_{\min}) / z_0, \text{ where}$$

$$\tau_0 = \frac{\tau_1}{\sqrt{1 + a_1^2}} \text{ and } z_0 = \pi \tau_0^2 / D_\nu$$



❖ Pulse compression by use of a QPM and a dispersive medium:



The modulation phase factor $\exp(j\zeta t^2)$ has $\zeta < 0$.

$$\tau_0 = \frac{\tau_1}{\sqrt{1 + a_1^2}} = \frac{\tau_1}{\sqrt{1 + \zeta^2 \tau_1^4}}$$

$$z_{\min} = -\frac{\pi \zeta}{D_\nu} \frac{\tau_1^4}{1 + \zeta^2 \tau_1^4}$$

If $a_1^2 >> 1$, $\tau_0 \approx \frac{\tau_1}{a_1} = \frac{1}{\zeta \tau_1}$ and

$$z_{\min} \approx f = \frac{\pi}{-\zeta D_\nu} \quad \text{(focal length)}$$