$$\nabla\times\bar{E} = -j\omega\bar{B} - \bar{J}_m$$
$$\nabla\times\bar{H} = j\omega\bar{D} + \bar{J}$$
$$\nabla\cdot\bar{D} = \rho$$
$$\nabla\cdot\bar{B} = \rho_m$$

SIMPLE DUALITY

$\rho \to \rho_m$ $\quad$ $\rho_m \to \rho$

$\bar{D} \to \bar{B}$ $\quad$ $\bar{B} \to \bar{D}$

$\bar{E} \to -\bar{H}$ $\quad$ $\bar{H} \to -\bar{E}$

$\bar{J}_m \to \bar{J}$ $\quad$ $\bar{J} \to \bar{J}_m$

Duality: $\bar{E}_d = \alpha\bar{H}$

$\bar{H}_d = \beta\bar{E}$

$$\nabla\times\bar{E}_d = -j\omega\bar{B}_d - \bar{J}_{md} \Rightarrow \alpha\nabla\times\bar{H} = -j\omega\bar{B}_d - \bar{J}_{md}$$
$$= \alpha(\bar{J} + j\omega\bar{D})$$

$\bar{B}_d = -\alpha\bar{D}$

$\bar{J}_{md} = -\alpha\bar{J}$

$$\nabla\times\bar{H}_d = j\omega\bar{D}_d + \bar{J}_d$$
$$= \beta\,\nabla\times\bar{E} = \beta(-\bar{J}_m - j\omega\bar{B})$$

$\bar{D}_d = -\beta\bar{B}$

$\bar{J}_d = -\beta\bar{J}_m$

$$\nabla\cdot\bar{D}_d = \rho_d$$
$$= -\beta\,\nabla\cdot\bar{B} = -\beta\rho_m$$

$\rho_d = -\beta\rho_m$

$$\nabla\cdot\bar{B}_d = \rho_{md}$$
$$= -\alpha\,\nabla\cdot\bar{D} = -\alpha\rho$$

$\rho_{md} = -\alpha\rho$

$$\bar{D} = \bar{\bar{\varepsilon}}\cdot\bar{E}_d + \bar{\bar{\xi}}_d\cdot\bar{H}_d$$

$$\bar{D}_d = \bar{\bar{\varepsilon}}_d \cdot \bar{E}_d + \bar{\bar{\xi}}_d \cdot \bar{H}_d$$

$$-\beta \bar{B} = \bar{\bar{\varepsilon}}_d \cdot \alpha \bar{H} + \bar{\bar{\xi}}_d \cdot \beta \bar{E}$$

$$\bar{B} = \underbrace{-\bar{\bar{\xi}}_d}_{\bar{\bar{\zeta}}} \cdot \bar{E} \underbrace{- \frac{\alpha}{\beta} \bar{\bar{\varepsilon}}_d}_{\bar{\bar{\mu}}} \cdot \bar{H}$$

$$\bar{B}_d = \bar{\bar{\zeta}}_d \cdot \bar{E} + \bar{\bar{\mu}}_d \cdot \bar{H}$$

$$= -\alpha \bar{D} = \alpha \bar{\bar{\zeta}}_d \cdot \bar{H} + \beta \bar{\bar{\mu}}_d \cdot \bar{E}$$

$$\bar{\bar{\varepsilon}}_d = -\frac{\beta}{\alpha} \bar{\bar{\mu}}$$

$$\bar{\bar{\xi}}_d = -\bar{\bar{\zeta}}$$

$$\bar{\bar{\zeta}}_d = -\bar{\bar{\xi}}$$

$$\bar{\bar{\mu}}_d = -\frac{\alpha}{\beta} \bar{\bar{\varepsilon}}$$

① INVOLUTION

$$(\underbrace{\bar{E}_d}_{\alpha \bar{H}})_d = \bar{E}$$

$$\bar{H}_d = \beta \bar{E}$$

$$\alpha \bar{H}_d = \alpha\beta \bar{E} = \bar{E}$$

$$\alpha\beta = +1$$

$$\beta = \frac{1}{\alpha}$$

② $(\varepsilon_0, \mu_0)_d = \varepsilon_0, \mu_0$

$$\varepsilon_d = -\frac{\beta}{\alpha}\mu$$

$$\mu_d = -\frac{\alpha}{\beta}\varepsilon$$

$$\varepsilon_{0_d} = -\frac{\beta}{\alpha}\mu_0 = \varepsilon_0$$

$$\mu_{0d} = -\frac{\alpha}{\beta}\varepsilon_0 = \mu_0$$

$$(\varepsilon_{0d}\,\mu_{0d} = \varepsilon_0\mu_0)$$

$$\Rightarrow \quad \frac{\beta}{\alpha} = -\frac{\varepsilon_0}{\mu_0}$$

$$\frac{\alpha}{\beta} = -\eta_0^2$$

$$\frac{\alpha}{\beta} = \alpha^2 = -\eta_0^2$$

$$\alpha = \pm j\eta_0$$

$$\bar{E}_d \times \bar{H}_d^* = \alpha \bar{H} \times (\beta \bar{E})^*$$

$$= \pm j\eta_0 \bar{H} \times \left(\pm \frac{j}{\eta_0} \bar{E}^* \right)$$

$$= - \bar{H} \times \bar{E}^*$$

$$= \bar{E}^* \times \bar{H}$$

(Real part remains)
invariant

$$\alpha = \pm j\eta_0$$

$$\beta = \mp \frac{j}{\eta_0}$$

$$\beta = \frac{1}{\alpha} = \frac{1}{\pm j\eta_0}$$

$$= \pm \frac{1}{j\eta_0} = \mp \frac{j}{\eta_0}$$

$$\varepsilon_{rd} \longrightarrow \mu_r$$

$$\varepsilon_d = \varepsilon_{rd}\,\varepsilon_0 = -\frac{\beta}{\alpha}\mu = -\left(-\frac{1}{\eta_0^2}\right)\mu$$

$$= \frac{\varepsilon_0}{\cancel{\mu_0}} \mu_r \cancel{\mu_0}$$

$$\boxed{\varepsilon_{rd} = \mu_r}$$

$$\text{PEC}_d = \text{PMC}$$

σ large

$$\varepsilon = \varepsilon' - j\sigma/\omega$$

$$k_d = \omega\sqrt{\mu_d \varepsilon_d} = \omega\sqrt{\mu\varepsilon} = k$$

$$\eta_d = \sqrt{\frac{\mu_d}{\varepsilon_d}} = \sqrt{\frac{\eta_0^2\,\varepsilon}{\mu}\eta_0^2}$$

$$= \eta_0^2 \frac{1}{\eta}$$

$$\eta_d \eta = \eta_0^2$$

$$\varepsilon_d = \frac{\mu}{\eta_0^2}$$

$$\mu_d = -\frac{\alpha}{\beta}\varepsilon$$

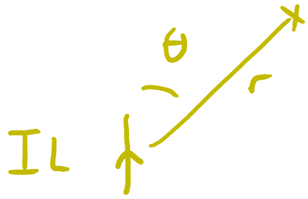
$$= \eta_0^2 \varepsilon$$

$\bar{\eta}$

∂ ~~ρ~~ + $\bar{J} = I L \bar{u}\, \delta(\bar{r})$

dual: $\bar{J}_d = -\beta \bar{J}_m = \frac{j}{\eta_0} \bar{J}_m$

$\bar{J}_m = -j\eta_0 \bar{J}_d$ $\quad \frac{j}{\eta_0} I_m L \bar{u}\, \delta(\bar{r})$



$$\bar{E}_d(\bar{r}) = j\omega\mu_{o_d} I_d L \frac{e^{-jk_d r}}{4\pi r}\sin\theta\ \bar{u}_\theta$$

$$IL = \int \mathcal{J}\, dv$$

$$\alpha\bar{H}(\bar{r}) = j\omega\mu_0\left(-\beta I_m L\right)\frac{e^{-jkr}}{4\pi r}\sin\theta\ \bar{u}_\theta$$

$$\bar{H}(\bar{r}) = -j\omega\mu_0\frac{\beta}{\alpha} I_m L\frac{e^{-jkr}}{4\pi r}\sin\theta\ \bar{u}_\theta$$

$$= +j\underbrace{\omega\mu_0\frac{1}{\eta_0^2}}_{\varepsilon_0} I_m L\frac{e^{-jkr}}{4\pi r}\sin\theta\ \bar{u}_\theta$$

Self-duality

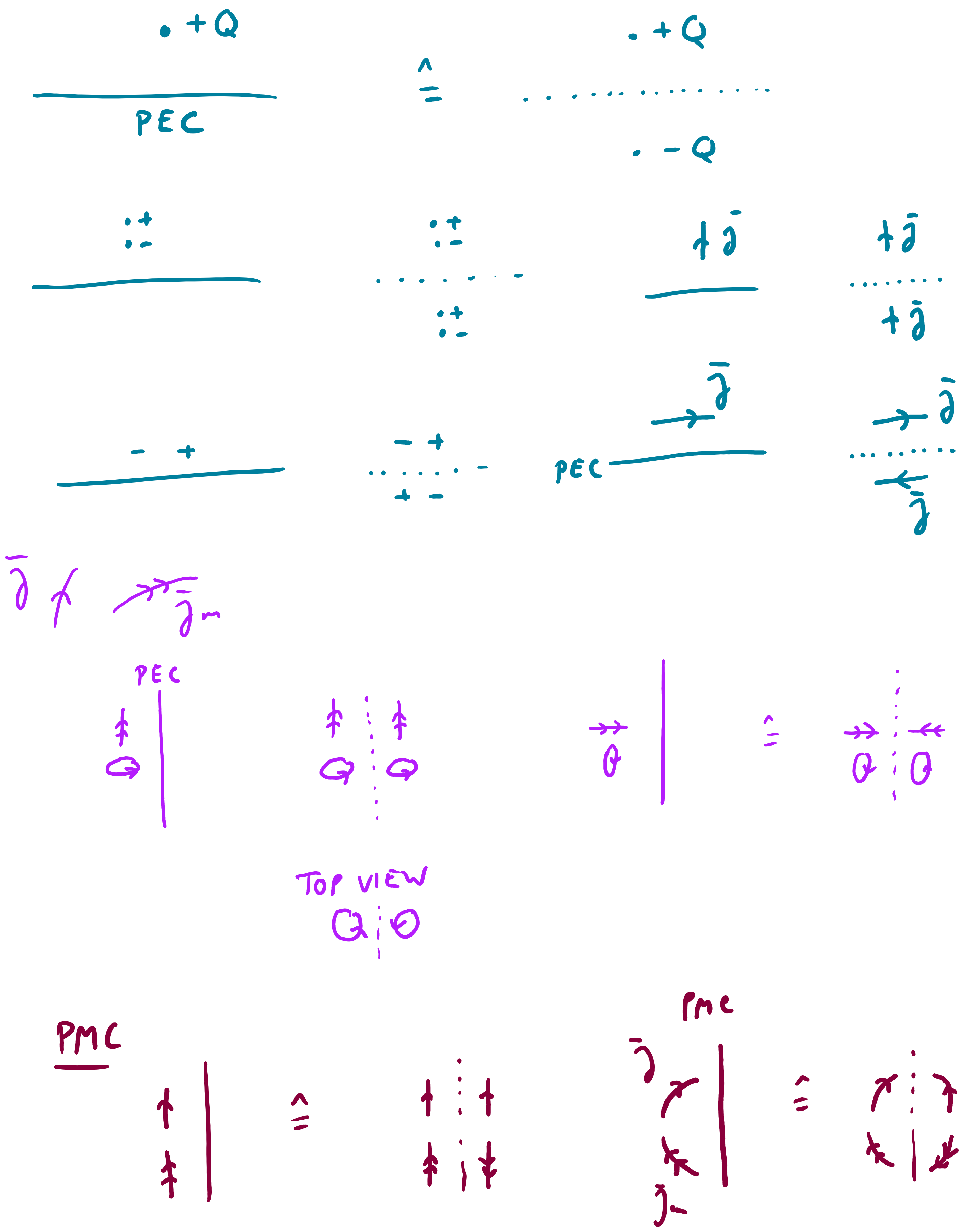
$\bar{E}, \bar{H}$

$$\bar{E}_{\pm} = \frac{1}{2}\left(\bar{E} \mp j\eta_0 \bar{H}\right)$$

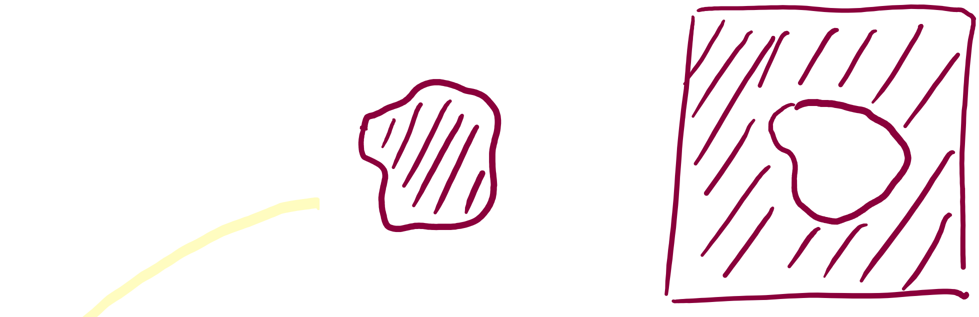
$$\bar{E}_{+d} = \frac{1}{2}\left(\alpha \bar{H} - j\eta_0 \beta \bar{E}\right)$$

$$= \frac{1}{2}\left(-j\eta_0 \bar{H} - j\eta_0 \frac{1}{-j\eta_0} \bar{E}\right) = \bar{E}_{+}$$

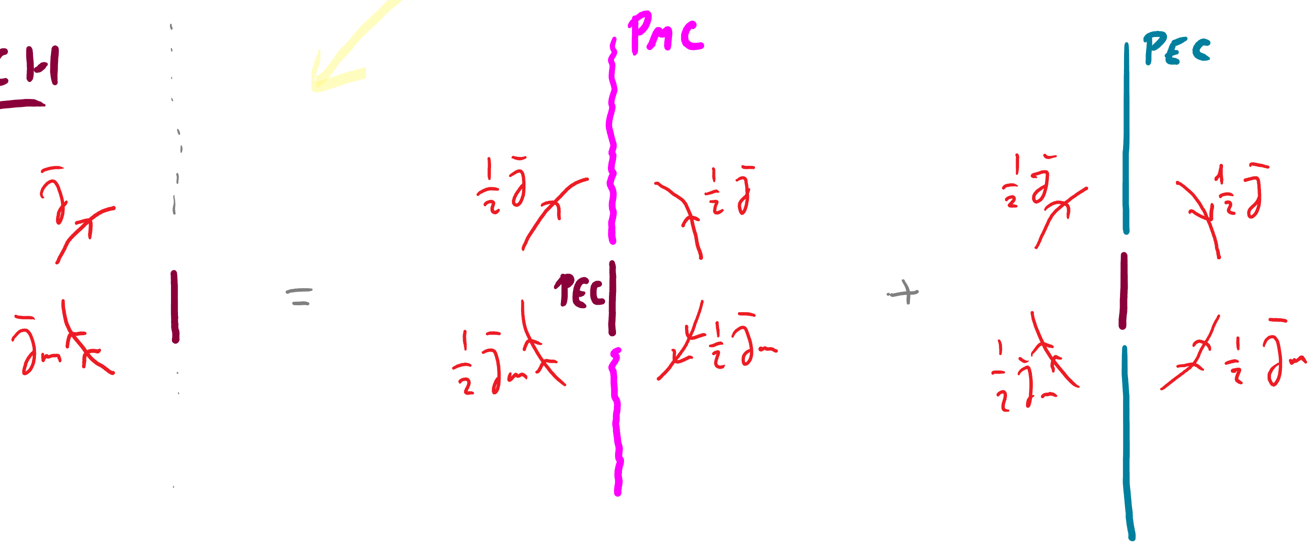
$$\bar{E}_{\pm} \cdot \bar{E}_{\pm} = 0$$


+Q
PEC
+Q
−Q
PEC
PEC
TOP VIEW
PMC
PMC

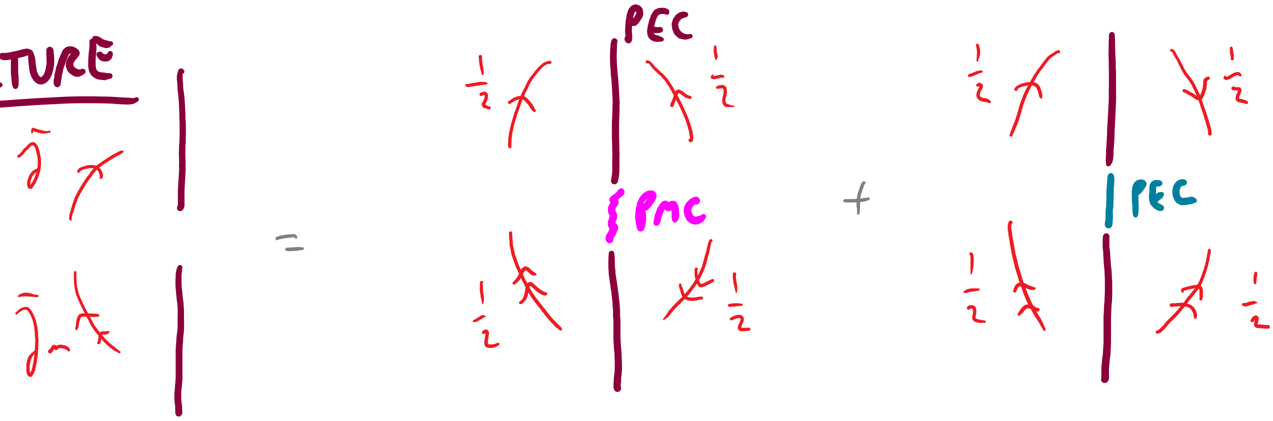
BABINET



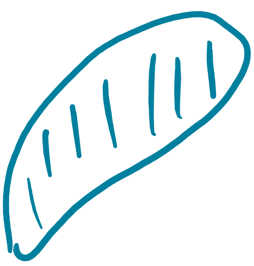
PATCH



APERTURE

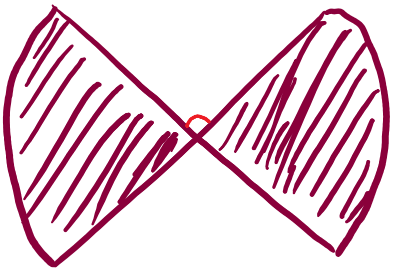


SELF-COMPLEMENTARY ANTENNAS



$$Z_d Z = \left(\frac{\eta_0}{2}\right)^2$$

$$Z_d = Z \Rightarrow \underline{189\,\Omega}$$



BOWTIE ANTENNA