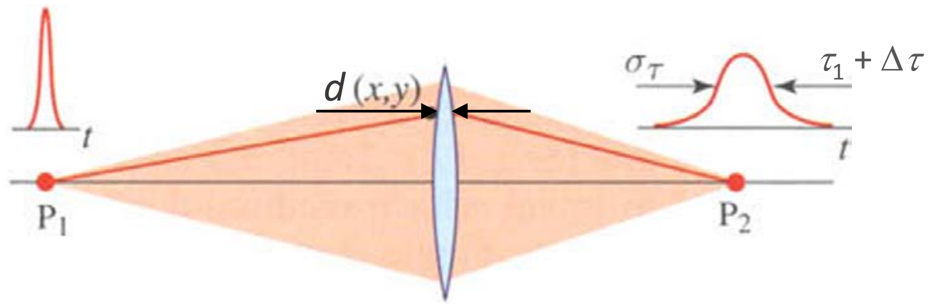
Chapter 22

ULTRAFAST OPTICS II

Ultrafast linear optics

Pulse broadening in a single-lens imaging system:

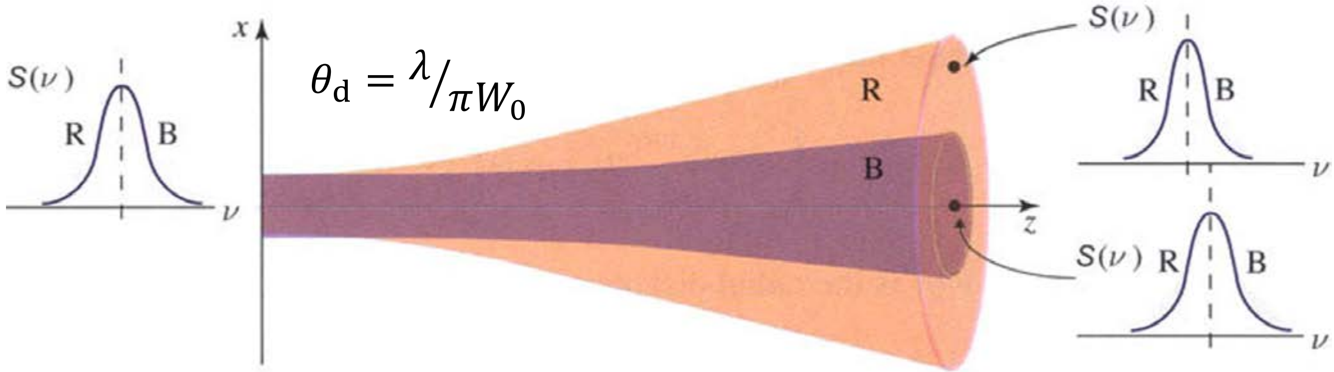


Time delay due to propagation in glass is $\Delta\tau(x,y) = \left|\frac{1}{c} - \frac{1}{v}\right| d(x,y) = \frac{|n-N|}{c_o} d(x,y)$

$$\Rightarrow \Delta\tau = \frac{|n-N|}{n-1} \frac{1}{8F_{\#}^2} \frac{f}{c_o},$$

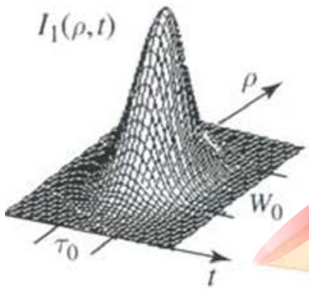
where $F_{\#} = f/D$ is the lens F-number.

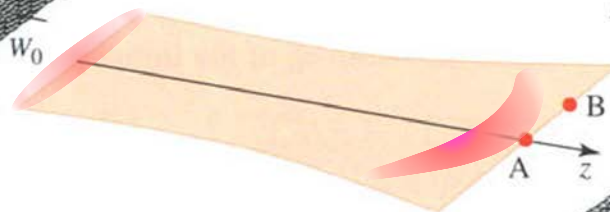
Transverse spectral spreading of a pulsed beam:

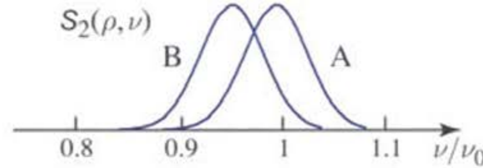


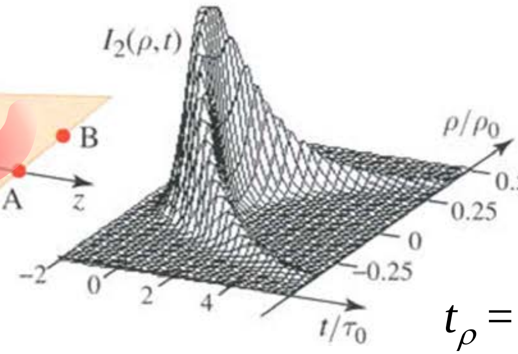
Monochromatic plane-wave decomposition (spatiotemporal FT approach) can be used.

Applying *Fourier optics* to a *Gaussian-pulsed Gaussian beam* that propagates in free space, one obtains:









$W(z) = W_0\, z/z_0$,

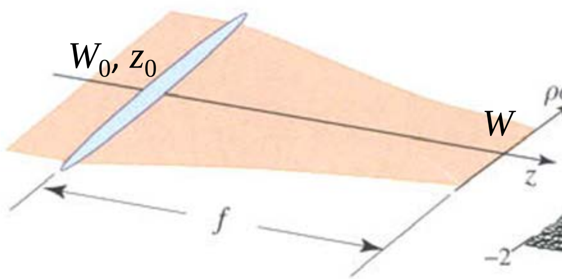
$\rho_0 = \pi N W(z)$, $\quad N = \nu_0 \tau_0$,

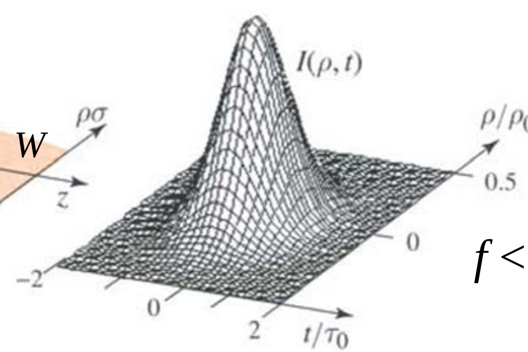
$t_\rho = t - \rho^2/2cz$

$$I_2(x,y,t) \propto \frac{\exp[-2\pi N\rho^2/(\rho^2+\rho_0^2)]}{1+\rho^2/\rho_0^2}\,\frac{\exp(-2t_\rho^2/\tau_\rho^2)}{1+t_\rho^2/\pi^2N^2\tau_0^2}, \qquad \tau_\rho = \tau_0\sqrt{1+\rho^2/\rho_0^2}$$

$$S_2(x,y,\nu) \propto \frac{\nu^2}{\nu_0^2}\exp\left[-2\pi^2N^2\frac{\rho^2}{\rho^2+\rho_0^2}\right]\exp\left[-2\pi^2N^2\frac{(\nu-\nu_\rho)^2}{\nu_\rho^2}\right], \qquad \nu_\rho = \frac{\nu_0}{1+\rho^2/\rho_0^2}$$

If the pulsed beam is *focused* with a nondispersive lens, the above equations are valid with $z = f$.





$f << \rho_0$

Ultrafast nonlinear optics

The conditions for pulsed *three-wave mixing* in a *second-order nonlinear medium* are

$\omega_1 + \omega_2 = \omega_3$, $\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3$, and $v_1 = v_2 = v_3$ (group velocities)

If $\beta(\omega_q + \Omega) \approx \beta_q + \Omega\beta'_q$, q = 1, 2, 3, the coupled-wave equations are

$$\beta'_q = \frac{1}{v_q} \qquad \left(\frac{\partial}{\partial z} + \frac{1}{v_1}\frac{\partial}{\partial t}\right) a_1 = -jga_3a_2^*$$

$$\beta_q \to \frac{\partial}{\partial z} \qquad \left(\frac{\partial}{\partial z} + \frac{1}{v_2}\frac{\partial}{\partial t}\right) a_2 = -jga_3a_1^*$$

$$\Omega \to \frac{\partial}{\partial t} \qquad \left(\frac{\partial}{\partial z} + \frac{1}{v_3}\frac{\partial}{\partial t}\right) a_3 = -jga_1a_2$$

new terms compared to CW case

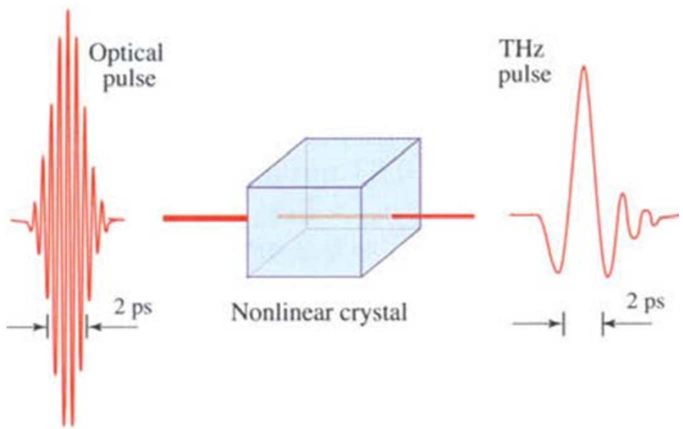
If $\beta(\omega_q + \Omega) \approx \beta_q + \Omega\beta'_q + \frac{1}{2}\Omega^2\beta''_q$ (GVD), the coupled-wave equations are

$$\left(\frac{\partial}{\partial z} + \frac{1}{v_1}\frac{\partial}{\partial t} - j\frac{\beta''_1}{2}\frac{\partial^2}{\partial t^2}\right) a_1 = -jga_3a_2^*$$

$$\left(\frac{\partial}{\partial z} + \frac{1}{v_2}\frac{\partial}{\partial t} - j\frac{\beta''_2}{2}\frac{\partial^2}{\partial t^2}\right) a_2 = -jga_3a_1^*$$

$$\left(\frac{\partial}{\partial z} + \frac{1}{v_3}\frac{\partial}{\partial t} - j\frac{\beta''_3}{2}\frac{\partial^2}{\partial t^2}\right) a_3 = -jga_1a_2.$$

❖ **THz pulse generation by down-conversion** (optical rectification)



Monochromatic components of the pulse, $\omega_1 = \omega$ and $\omega_2 = \omega + \Omega$, are mixed in pairs

$$\Rightarrow P_{\mathrm{THz}}(\Omega) = \int 2\mathrm{d}E^*(\omega)E(\omega+\Omega)d\omega.$$

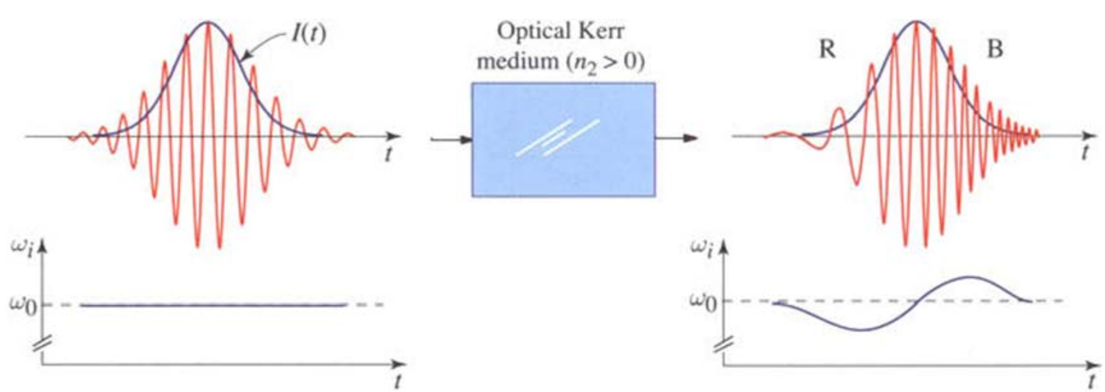
The phase-matching error determines the frequency-conversion length $L_c = 2\pi/|\Delta k|$,

where $\Delta k = k(\omega+\Omega) - k(\omega) - k(\Omega) \approx [N(\omega) - n(\Omega)]\Omega/c_o.$

❖ **Pulse self-phase modulation (SPM)**

In a *third-order nonlinear medium*, the phase shift due to the optical Kerr effect is now

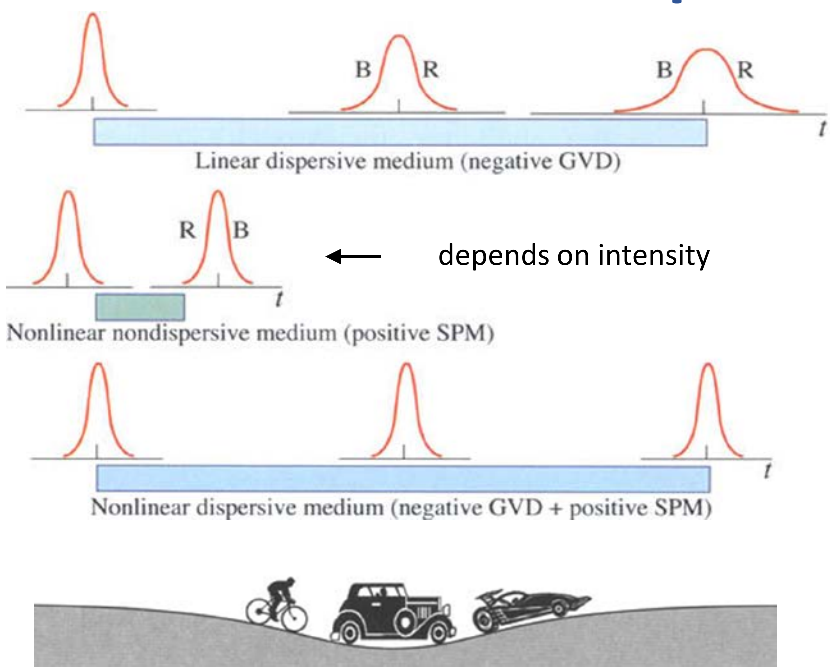
$$\Delta\varphi(t) = -n_2 I(t) k_0 z$$
$$\Downarrow$$
$$\Delta\omega_i = -n_2 \frac{dI}{dt} k_0 z$$
$$\Downarrow$$



The pulse is *up-chirped*. If near the center, we write $I \approx I_0(1\text{-}2t^2/\tau^2)$, the chirp parameter can be written as $a = z/z_{\mathrm{NL}}$, where

$$z_{\mathrm{NL}} = (2n_2 I_0 k_0)^{-1} \quad \text{(nonlinear characteristic length)}$$

Optical solitons



The chirp factor due to SPM must be equal to the negative of that due to GVD:

$$2n_2 I_0 k_0 \Delta z = -2\beta'' \tau_0^{-2} \Delta z$$

$$\Rightarrow \boxed{k_0 n_2 I_0 = -\frac{\beta''}{\tau_0^2}} = (2z_0)^{-1}$$

Shorter pulses require higher intensities.

The soliton envelope must be given by

$$\boxed{\mathcal{A}(z,t) = \mathcal{A}_0 \operatorname{sech}\left(\frac{t - z/v}{\tau_0}\right) \exp\left(jz/4z_0\right),}$$

which is a solution of the wave equation in the presence of GVD and SPM. In general,

$$\left[\nabla^2 - \frac{1}{c_o^2}\frac{\partial^2}{\partial t^2}\right]\mathcal{E} = \mu_o \frac{\partial^2}{\partial t^2}\left(\mathcal{P}_L + \mathcal{P}_{\rm NL}\right) \Rightarrow \left[\nabla^2 + \beta^2(\omega)\right]\mathcal{E} = -\mu_o \omega^2 \mathcal{P}_{\rm NL}$$

In the slowly varying envelope approximation at weak dispersion and nonlinearity,

$$\mathcal{E} = \mathrm{Re}\{\mathcal{A}(z,t)\exp[j(\omega_0 t - \beta_0 z]\} \Rightarrow \frac{D_\nu}{4\pi}\frac{\partial^2 \mathcal{A}}{\partial t^2} + \gamma|\mathcal{A}|^2\mathcal{A} + j\left(\frac{\partial}{\partial z} + \frac{1}{v}\frac{\partial}{\partial t}\right)\mathcal{A} = 0$$

Dimensionless variables:

$$\mathrm{t} = \frac{t - z/v}{\tau_0}, \quad \mathrm{z} = \frac{z}{2z_0}, \quad \psi = \frac{\mathcal{A}}{\mathcal{A}_0} \quad \Rightarrow$$

Nonlinear Schrödinger equation

$$\boxed{\frac{1}{2}\frac{\partial^2 \psi}{\partial \mathrm{t}^2} + |\psi|^2 \psi + j\frac{\partial \psi}{\partial \mathrm{z}} = 0}$$

Spatial and temporal solitons

In the moving frame, the Schrödinger equation is

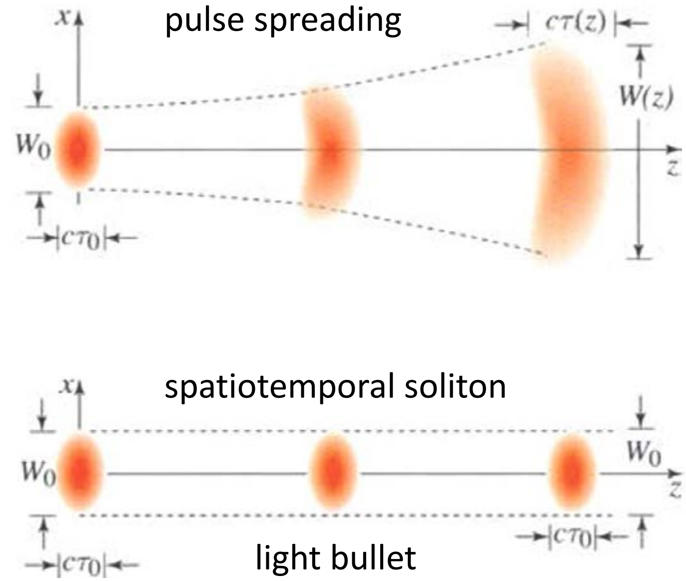
$$\frac{D_\nu}{4\pi}\frac{\partial^2 A}{\partial t^2} + \gamma|A|^2 A + j\frac{\partial A}{\partial z} = 0, \quad \gamma = \frac{\pi n_2}{\lambda \eta_0}$$

Nonlinear 1D beam diffraction was described by

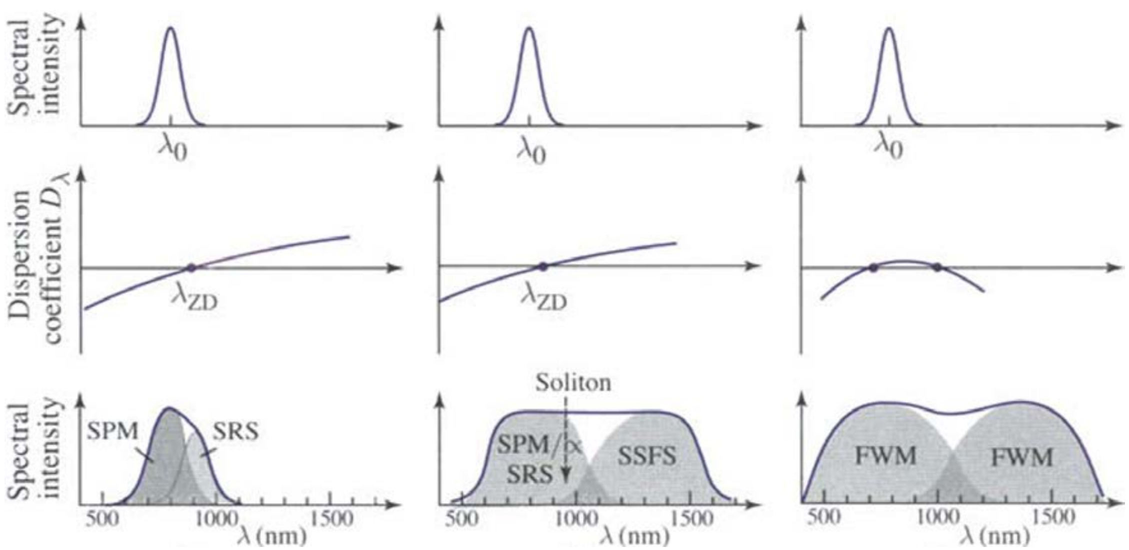
$$-\frac{\lambda}{4\pi}\frac{\partial^2 A}{\partial x^2} + \gamma|A|^2 A + j\frac{\partial A}{\partial z} = 0$$

Nonlinear diffraction-dispersion equation is

$$-\frac{\lambda}{4\pi}\nabla_T^2 A + \frac{D_\nu}{4\pi}\frac{\partial^2 A}{\partial t^2} + \gamma|A|^2 A + j\frac{\partial A}{\partial z} = 0.$$



Supercontinuum light generation in a fiber



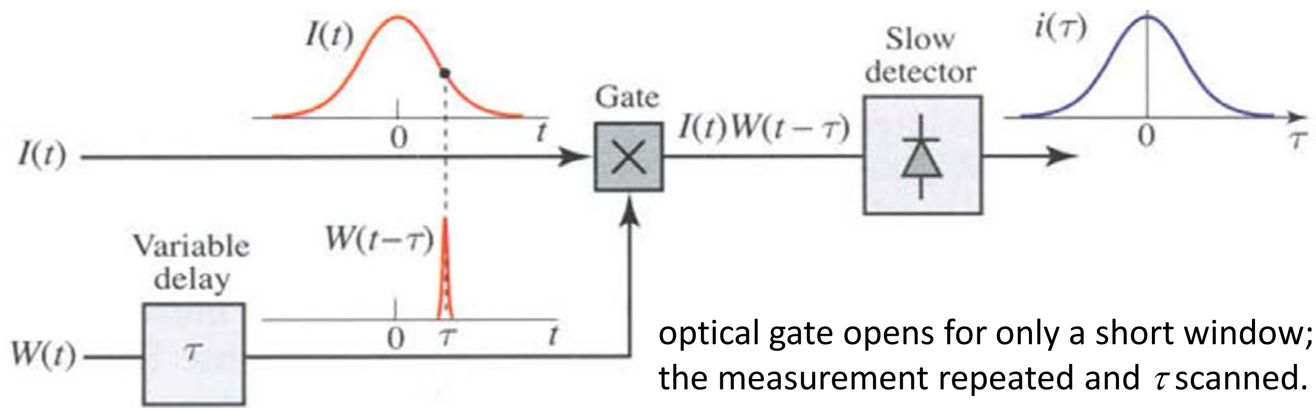
Broadening mechanisms:

Self-phase modulation
($\Delta\omega_0\sqrt{1+a^2}$)
Stimulated Raman scattering
(red-shift due to vibrations)
Soliton self-frequency shift
(due to intrapulse SRS)
Four-wave mixing
(2 peaks broadened by SPM)

Pulse detection

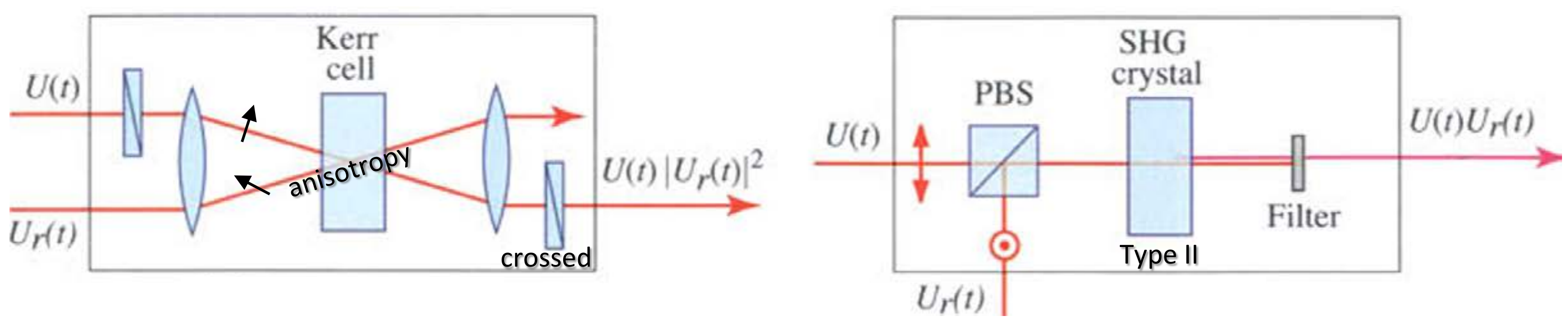
Fastest photodetectors have a response time of 100 ps and are too slow for fs pulses.

⇒ Slow detector + fast shutter:

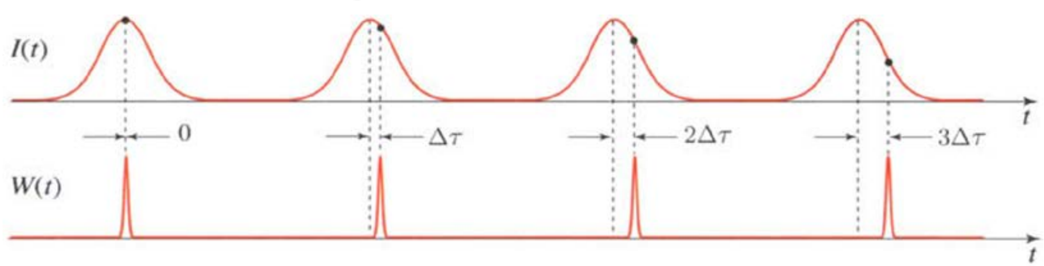


optical gate opens for only a short window;
the measurement repeated and τ scanned.

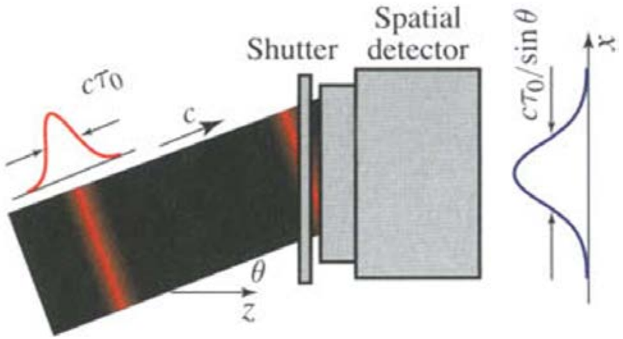
Gate realizations:

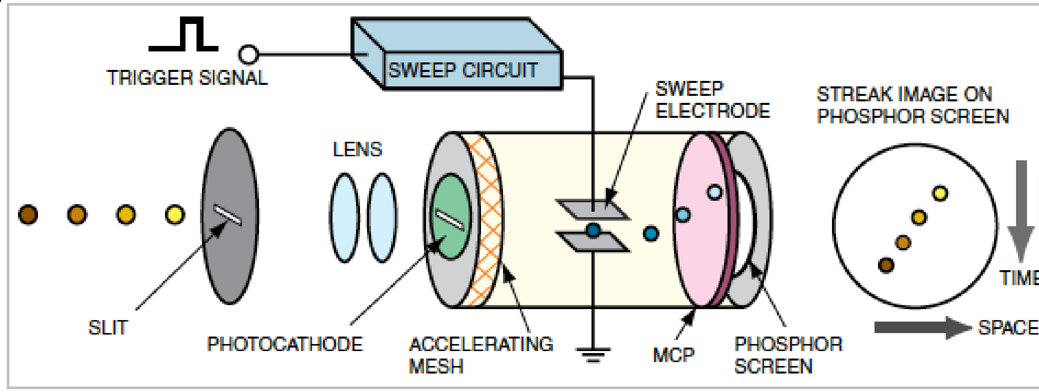


Pulse-train or multiple-detector ***single-shot** measurement*:

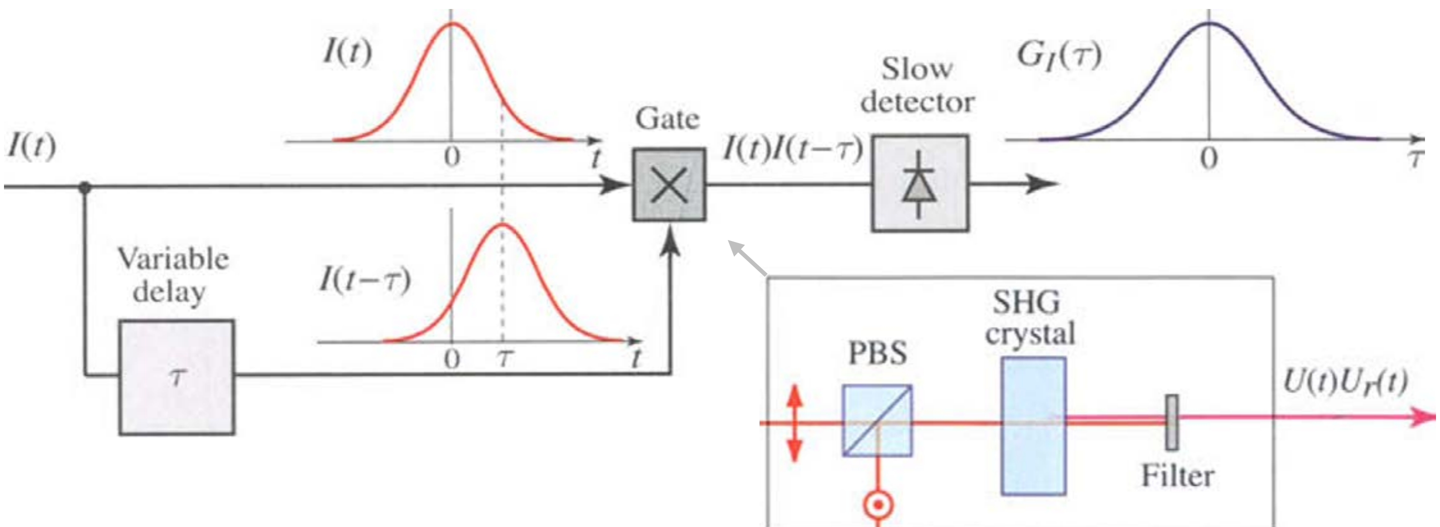


Streak camera (100 fs resolution)





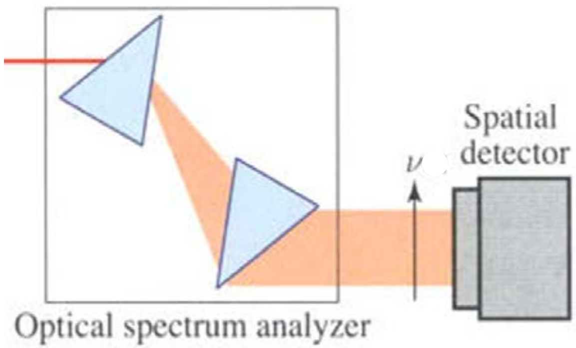
Intensity autocorrelation measurement (the pulse provides the gate to itself)



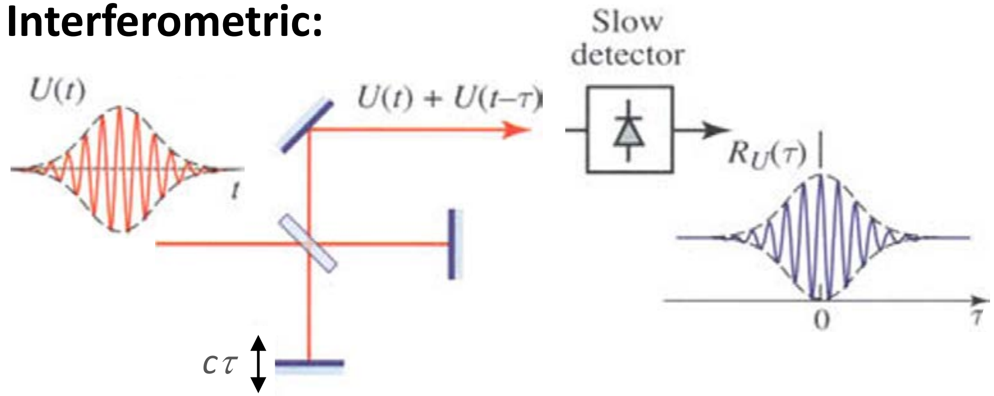
The intensity autocorrelation is $G_I(\tau) = \int I(t)I(t-\tau)dt$. For example, if the signal is $I(t) = \exp(-2\tau^2/\tau_0^2)$, the autocorrelation is $G_I(\tau) \propto \exp(-\tau^2/\tau_0^2)$, which gives τ_0.

Spectrum analyzers

Ordinary:



Interferometric:



$$R_U(\tau) = G_A(0) + \mathrm{Re}\left\{G_A(\tau)\exp(-j2\pi\nu_0\tau)\right\},\quad S(\nu) = \mathrm{FT}\{G_A(\tau)\}$$

$$G_A(\tau) = \int A^*(t)A(t-\tau)dt$$

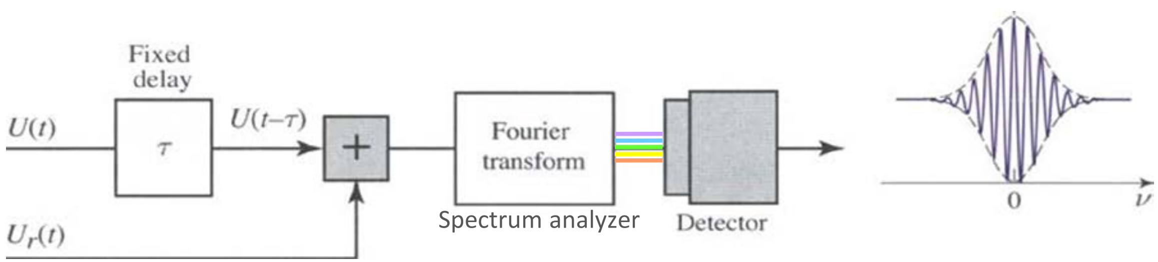
Measurement of phase

Heterodyning (time-domain interference with a known field) could reveal the phase, but only if it varies slowly:

$$|U(t) + U_r(t)|^2 = I(t) + I_r(t) + 2\sqrt{I(t)I_r(t)}\cos\left[2\pi f t + \varphi_r(t) - \underline{\varphi(t)}\right]$$

Spectral interferometry, in contrast, generates interferograms in the Fourier domain:

$$\left|V(\nu)e^{-j2\pi\tau\nu} + V_r(\nu)\right|^2 = S(\nu) + S_r(\nu) + 2\sqrt{S(\nu)S_r(\nu)}\cos\left[2\pi\tau\nu + \psi_r(\nu) - \psi(\nu)\right]$$



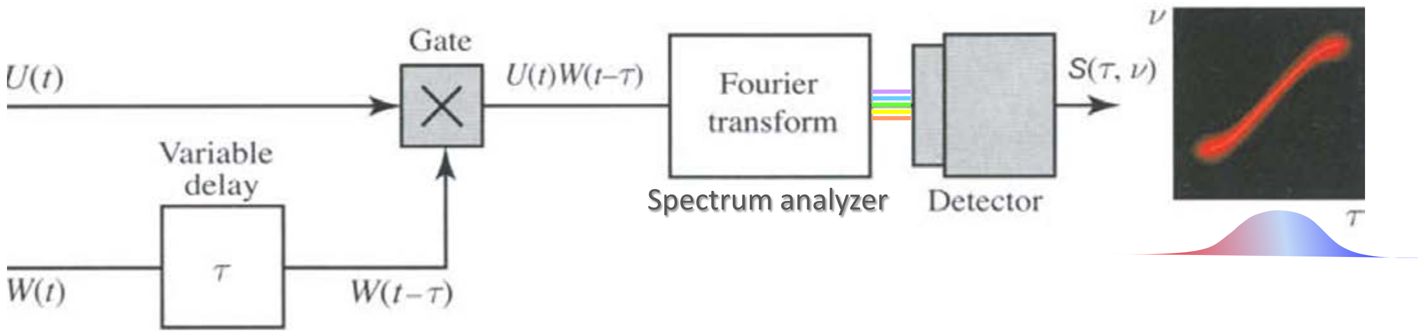
$\Rightarrow$ $\underline{\varphi(t)}$

Measurement of spectrogram

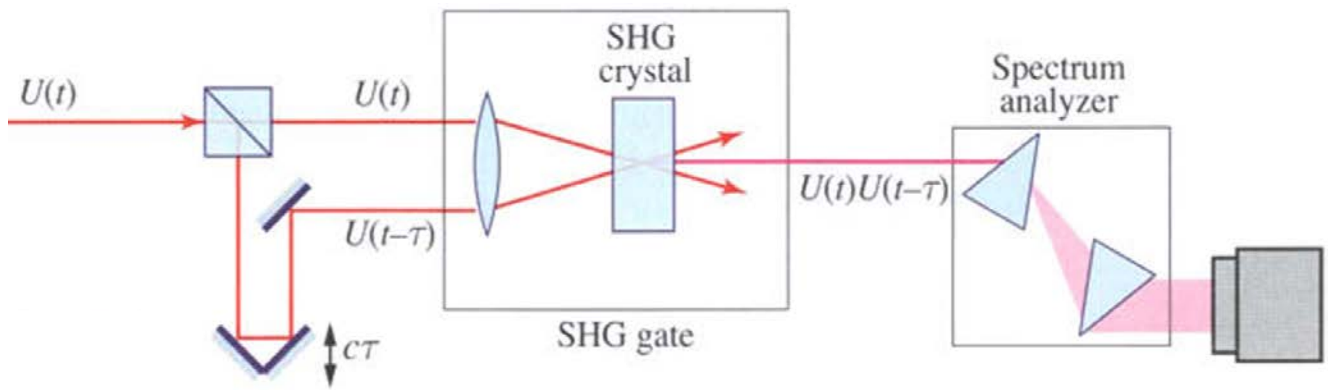
The spectrum of a *windowed part* of the measured pulse is

$$S(\nu,\tau) = |\Phi(\nu,\tau)|^2; \quad \Phi(\nu,\tau) = \int U(t)W(t-\tau)\exp(-j2\pi\nu t)dt.$$

The measurement technique is known as **frequency-resolved optical gating** (FROG):



Experimental implementation of a SHG-FROG [with $W(t) = U(t)$]:



The phase of $\Phi(\nu, \tau)$ is not measured, but $U(t)$ can still be retrieved using certain iteration algorithms. Otherwise, we could find $U(t) \propto \iint \Phi(\nu,\tau)\exp(j2\pi\nu t)\,d\nu d\tau$.