Aalto University
School of Science

PHYS-E0421 Solid State Physics
Period V, spring 2019

Martti Puska
Rina Ibragimova

Hannu-Pekka Komsa
Arsalan Hashemi

Dielectric Properties of Solids
Magnetism

Lecture 13, Monday 13.5.2019

Magnetic properties

- Response of materials to an external magnetic field
  - Magnetic quantities, magnetism is quantum mechanics (home work)
  - Quantum mechanical description
  - Atomic diamagnetism, paramagnetism (lecture work)
  - Response of free electron gas

- Spontaneous magnetism Ferromagnetism and antiferromagnetism
  - Exchange interaction, H_2 molecule, Heisenberg spin Hamiltonian
  - Mean-field approximation for ferromagnetism of magnetic moments
  - Spin waves (low-energy excitations) TODAY
  - Free electron gas
  - Stoner model for ferromagnetism of itinerant electrons
  - Antiferromagnetism
  - Domain structure

Spontaneous Magnetization, Starting Point

Ferromagnetism, $T < \theta_{CW}$

$\mathbf{B}_0 \to 0$ → $\mathbf{M} \neq 0$

Ordering

$\boldsymbol{\mu}_i \uparrow\uparrow\uparrow\uparrow\uparrow \boldsymbol{\mu}_j$

Coulomb interaction

$U_{Coul.}(e^- \leftrightarrow e^-) \sim 0.1 \ldots 1\,\mathrm{eV}$

Pauli exclusion principle

→ Total-spin dependent U_{coul} = Exchange interaction → Ordering

Example, H_2 molecule

Independent electron model

→ The lowest-energy molecular orbital is occupied by two electrons of opposite spins (singlet state). Two electrons of paralled spins (triplet state) occupy both the bonding and antibonding orbitals.

Many-body wave function

→ Heitler-London approximation based on atomic orbitals.
Two-electron orbital wave functions for singlet and triplet states

$$\Phi_S(1,2) = \phi_\mathrm{a}(1)\phi_\mathrm{b}(2) + \phi_\mathrm{b}(1)\phi_\mathrm{a}(2) \quad ; \quad \Phi_T(1,2) = \phi_\mathrm{b}(1)\phi_\mathrm{a}(2) - \phi_\mathrm{a}(1)\phi_\mathrm{b}(2)$$

→ E_S-$E_T < 0$ ← Exchange interaction

Heisenberg Spin Hamiltonian

➔ Generalization to solids

H2 molecule

Desired
Schrödinger equation

$$\hat{H}_{Heis}\Theta_S = E_S\Theta_S$$
$$\hat{H}_{Heis}\Theta_T = E_T\Theta_T$$

Two-electron spin eigenfunctions

$$\Theta_S = \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle_1\left|\downarrow\right\rangle_2 - \left|\downarrow\right\rangle_1\left|\uparrow\right\rangle_2\right)$$

$$\Theta_T = \begin{cases} \left|\uparrow\right\rangle\left|\uparrow\right\rangle \\ \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle\left|\downarrow\right\rangle + \left|\downarrow\right\rangle\left|\uparrow\right\rangle\right) \\ \left|\downarrow\right\rangle\left|\downarrow\right\rangle \end{cases}$$

Operator

Weighted mean value

$$\hat{H}_{Heis} = \frac{1}{4}(E_S + 3E_T) - (E_S - E_T)\hat{\mathbf{S}}_1 \bullet \hat{\mathbf{S}}_2$$

Proof

$$\hat{\mathbf{S}} = \hat{\mathbf{S}}_1 + \hat{\mathbf{S}}_2 \quad ; \quad \hat{\mathbf{S}}^2 = \hat{\mathbf{S}}_1{}^2 + 2\hat{\mathbf{S}}_1 \bullet \hat{\mathbf{S}}_2 + \hat{\mathbf{S}}_2{}^2 \quad ; \quad \hat{\mathbf{S}}^2\Theta = S(S+1)\Theta \, ; \quad \hat{\mathbf{S}}_1{}^2\Theta = S_1(S_1+1)\Theta \quad \text{etc.}$$

Parametric form

$$\hat{H}_{Heis} = -J\,\hat{\mathbf{S}}_1 \bullet \hat{\mathbf{S}}_2$$

[E(7.217)]

Exchange constant *J*

$J > 0:$ Ferromagnetic coupling, $\uparrow\uparrow$, $E_T < E_S$

$J < 0:$ Antiferromagnetic coupling, $\uparrow\downarrow$, $E_S < E_T$

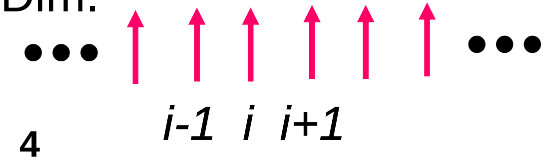
Excercise: Calculate $E_S = \left\langle \Theta_S \middle| \hat{H}_{Heis} \middle| \Theta_S \right\rangle$ and $E_T = \left\langle \Theta_T \middle| \hat{H}_{Heis} \middle| \Theta_T \right\rangle$

Exercise

Solids

$$\hat{H}_{Heis} = \sum_{\substack{i,j \\ \text{nearest neighbors}}} (\text{-}J_{ij})\hat{\mathbf{S}}_i \bullet \hat{\mathbf{S}}_j$$

1-Dim.

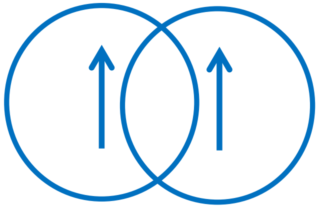


Exchange Interactions, different ranges

Direct Exchange

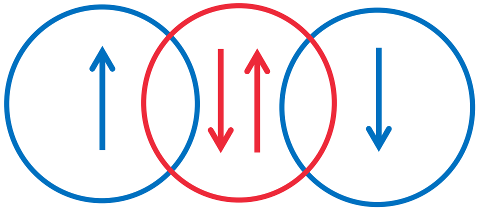
Direct interaction between two magnetic ions

Short-range



Super exchange:

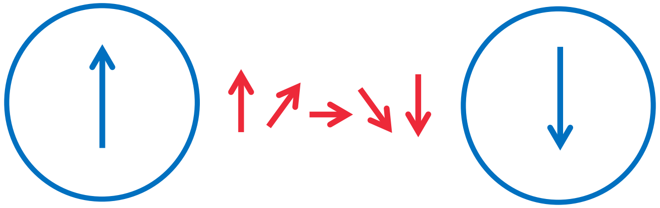
Interaction mediated via a non-magnetic ion



Example: CaMnO$_3$
Antiferromagnetic coupling
between two Mn ions via an O ion

Indirect exchange:

Interaction mediated via conducting electrons ↑



RKKY (Ruderman-Kittel-Kasuya-Yosida) distance-dependent interaction
Magnetic ion polarizes the electron gas and polarization oscillates with wave length λ
➔ Ferromagnetic or antiferromagnetic coupling

Ferromagnetism due to localized moments

(Elliott 7.2.5.2)

(e.g. 4*f* ions)

Heisenberg model

External field

$$\hat{H}_{Heis} = -\sum_{i} \sum_{\delta}^{\text{nearest neighbors}} J_{i\delta}\,\hat{\mathbf{S}}_i \bullet \hat{\mathbf{S}}_{i\delta} - g_J \mu_B \mathbf{B_0} \bullet \sum_{i} \hat{\mathbf{S}}_i$$

Nonlinear in $\mathbf{S}_i$

[~E(7.218)]

Exchange integral

> 0 ➔ Ferromagnetism

Total angular momentum operators

Magnetic moment

Mean-field approximation

Coordination number

Effective magnetic field, includes exchhange interaction

$$\hat{H}_{MF} = -\sum_{i} \hat{\mathbf{S}}_i \bullet \left(\sum_{\delta}^{\nu} J \left\langle \hat{\mathbf{S}}_{i\delta} \right\rangle_{ave} + g_J \mu_B \mathbf{B_0} \right) = -\sum_{i} g_J \mu_B \hat{\mathbf{S}}_i \bullet \mathbf{B}_{eff}$$

Expectation value $\langle \mathbf{S} \rangle$

Non-interacting gas of paramagnetic ions in field $\mathbf{B}_{eff}$

$$\mathbf{M} = \mathbf{M}_0(\mathbf{B}_{eff}, T)$$

e.g., $S = 1/2,\ g_J = 2:$

$$M_0(\mathbf{B}_{eff}, T) = \frac{N}{V}\mu_B \tanh(\mu_B B_{eff} / k_B T)$$

At every ion *i*

[~E(7.225)]

$$\langle \mathbf{S} \rangle = \frac{V}{N}\mathbf{M} \Big/ g_J \mu_B \Rightarrow \mathbf{B}_{eff} = \frac{1}{g_J \mu_B} \nu J \frac{V}{N} \mathbf{M} \Big/ g_J \mu_B + \mathbf{B_0}$$

The two unknowns $\mathbf{M}$ and $\mathbf{B}_{eff}$ should be solved selfconsistently

Mean-Field Theory of Ferromagnetism of Localized Moments

(Elliott 7.2.5.2)

Complementing the problem with $\mathbf{B}_{eff}$ and $\mathbf{M}_0$

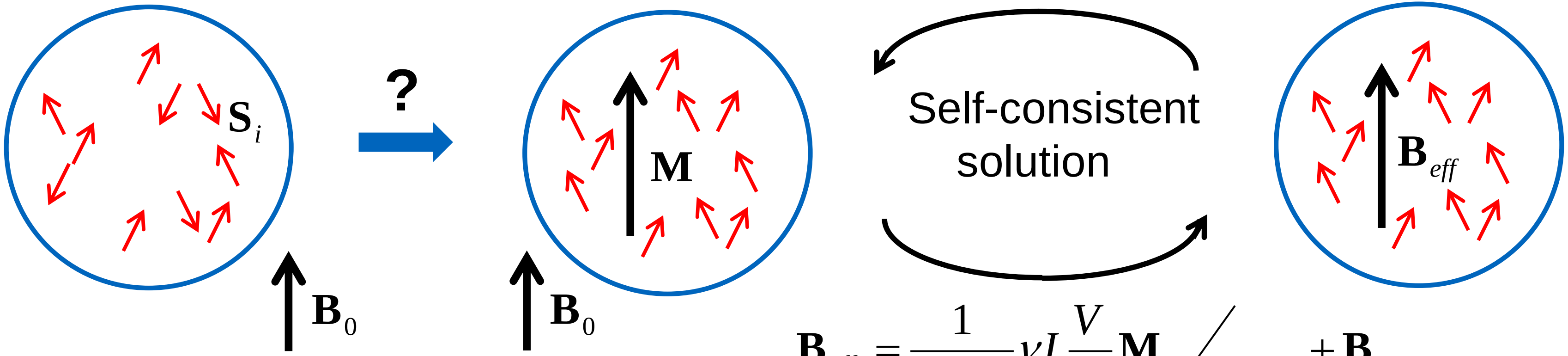
Temperature $T > 0$

Ordered system of paramagnetic ions with magnetization $\mathbf{M}$

[~E(7.222)]

$$\mathbf{M} = \mathbf{M}_0(\mathbf{B}_{eff}, T)$$

Noninteracting gas of paramagnetic ions in field $\mathbf{B}_{eff}$



Self-consistent solution

$$\mathbf{B}_{eff} = \underbrace{\frac{1}{g_J \mu_B} \nu J \frac{V}{N} \mathbf{M} \Big/ g_J \mu_B}_{} + \mathbf{B_0}$$

[~E(7.225)]

Caused by exchange interaction between magnetic moments

The two unknowns $\mathbf{M}$ and $\mathbf{B}_{eff}$ should be solved selfconsistently

1: When $\mathbf{B_0} = 0$ and T finite, is there a nontrivial (ferromagnetic) solution for $\mathbf{B}_{eff}$ and $\mathbf{M}$?

Ferromagnetism due to localized moments

➔ **1**. Do we have spontaneous $\mathbf{M} \neq 0$ at $\mathbf{B_0} = 0$ for temperature *T* ?

Self-consistency equations

$$\mathbf{M} = \mathbf{M}_0(\mathbf{B}_{eff}, T)$$

$$\mathbf{B}_{eff} = \frac{1}{g_J \mu_B} \nu J \frac{V}{N} \mathbf{M} \Big/ g_J \mu_B + \cancel{\mathbf{B_0}}$$

[~E(7.225)]

Assume the case

$$S = 1/2, \ g_J = 2,$$

[~E(7.227)]

$$M_0(\mathbf{B}_{eff}, T) = \frac{N}{V} \mu_B \tanh(\mu_B B_{eff} / k_B T)$$

Non-interacting paramagnetic ions

For $S = 1/2$

$$\boxed{\frac{N}{V} \mu_B = n\mu_B = M_{\max}}$$

$$\Rightarrow \begin{cases} M = \frac{N}{V} \mu_B \tanh(\mu_B B_{eff} / k_B T) = M_{\max} \tanh(\mu_B B_{eff} / k_B T) \\ B_{eff} = \frac{1}{4\mu_B{}^2} \nu J \frac{V}{N} M = \frac{\nu J}{4} \frac{V}{N\mu_B} \frac{1}{\mu_B} M = k_B \theta_{CW} \frac{1}{M_{\max}} \frac{1}{\mu_B} M \end{cases}$$

$$\boxed{k_B \theta_{CW} = \frac{\nu J}{4}}$$

Curie - Weiss temperature

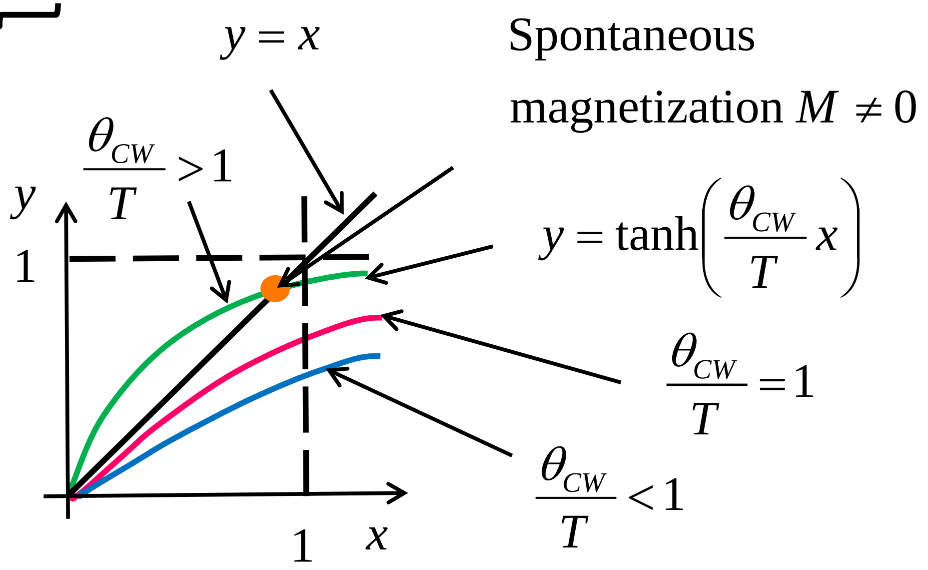
$$\Rightarrow \boxed{\frac{M}{M_{\max}} = \tanh\left(\frac{\theta_{CW}}{T} \frac{M}{M_{\max}} \right)}$$

Ferromagnetism due to localized moments

➔ 1. Do we have spontaneous $\mathbf{M} \neq 0$ at $\mathbf{B_0} = 0$ for temperature *T* ?

$$\boxed{\underbrace{\frac{M}{M_{\max}}}_{x} = \tanh\left(\frac{\theta_{CW}}{T}\underbrace{\frac{M}{M_{\max}}}_{x}\right)} = y(x)$$

$$\boxed{S = 1/2,\ g_J = 2}$$



$$\boxed{\tanh x \approx x - \frac{1}{3}x^3 \quad \text{for small } x}$$

$$\boxed{\begin{array}{l} T < \theta_{CW} \Rightarrow \text{Ferromagnetism} \\ T > \theta_{CW} \Rightarrow \text{Paramagnetism} \end{array}}$$

Critical temperatue
Curie-Weiss temperature

$$\boxed{\theta_{CW} = \nu J / 4k_B}$$

[~E(7.229)]

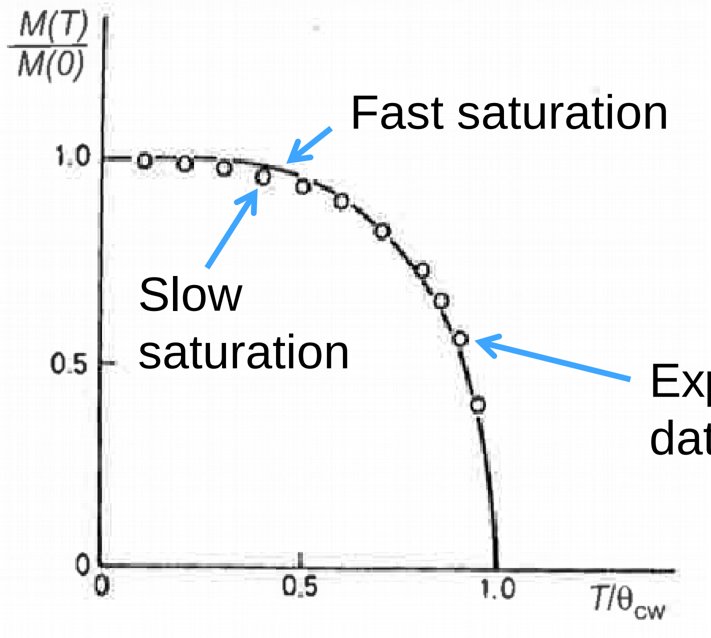
Coordination number

Strength of exchange interaction

Generalization to *S* >1/2

Mean-Field Theory *M(T)*, Different Temperature Regions

$$\boxed{\frac{M}{M_{\max}} = \tanh\left(\frac{M}{M_{\max}}\frac{\theta_{CW}}{T}\right)}$$



Fast saturation

Slow saturation

Experimental data for Ni

$\boxed{T << \theta_{CW}}$

$$M \approx M_{\max} = M(T=0)$$

$$\Rightarrow M / M_{\max} \approx \tanh(\theta_{CW} / T)$$

$$\tanh x \; \approx 1 - 2\exp(-2x) \quad \text{for large } x$$

Fast exponential saturation

$$\boxed{M/M_{\max} \approx 1 - 2\exp\left(-2\theta_{CW} / T\right)}$$

[~E(7.231)]

Experiment

$$\boxed{M/\mathrm{M}_{\max} \approx 1\text{-}AT^{3/2}}$$

← Spin waves

Slow power-law saturation

Mean-Field Theory *M(T)*, Different Temperature Regions

$\boxed{T \approx \theta_{CW}}$ ⇒ $\boxed{M << M_{\max}}$

$$\tanh x \approx x - \frac{1}{3}x^3 \quad \text{for small } x$$

$$\boxed{\frac{M}{M_{\max}} = \tanh\left(\frac{M}{M_{\max}}\frac{\theta_{CW}}{T}\right)}$$

$$\approx \frac{M}{M_{\max}}\frac{\theta_{CW}}{T} - \frac{1}{3}\left(\frac{M}{M_{\max}}\frac{\theta_{CW}}{T}\right)^3$$

$$\Rightarrow M/M_{\max} = \sqrt{3}\left(\frac{T}{\theta_{CW}}\right)^{3/2}\sqrt{\theta_{CW}/T - 1} = \sqrt{3}\,\overbrace{\left(T/\theta_{CW}\right)}^{\approx 1}\left(1 - T/\theta_{CW}\right)^{1/2}$$

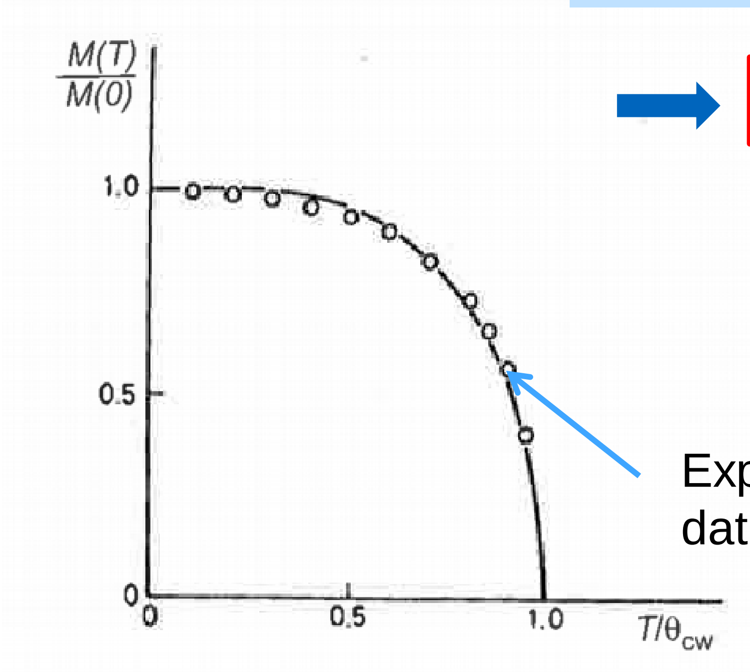
⇒ $\boxed{M/M_{\max} \approx \sqrt{3}\left(1 - T/\theta_{CW}\right)^{1/2}}$

[~E(7.232)]

Critical exponent

Experiment

$\boxed{M/M_{\max} \propto \left(1 - T/\theta_{CW}\right)^{1/3}}$



Experimental data for Ni

Mean-Field Theory *M(T)*, Different Temperature Regions

$\mathbf{2}: \quad T > \theta_{CW}$ → No spontaneous magnetization

$\mathbf{B_0} \neq 0$ → $\mathbf{M} \neq 0$

$\chi_m = ?$

Assume

$S = 1/2, \; g_J = 2$ →

Self-consistency equations

$$M = \frac{N}{V}\mu_B \tanh(\mu_B B_{eff}/k_B T)$$

$$\mathbf{B}_{eff} = \frac{1}{g_J \mu_B}\nu J \frac{V}{N}\mathbf{M} \Big/ g_J \mu_B + \mathbf{B_0}$$

$\tanh x \approx x$

$$= n\mu_B^2 B_{eff}/k_B T$$

$$= \frac{k_B \theta_{CW}}{\mu_B^2 n}\mathbf{M} + \mathbf{B_0}$$

$$k_B \theta_{CW} = \frac{\nu J}{4}$$

Curie-Weiss law

$$M = \frac{n\mu_B^2 B_0}{k_B(T - \theta_{CW})}$$

→

$$\chi_m = \frac{C}{T - \theta_{CW}}$$

[E(7.234)]

Cf. Curie law,
Contribution of exchange-interactions between ions!

Experiment

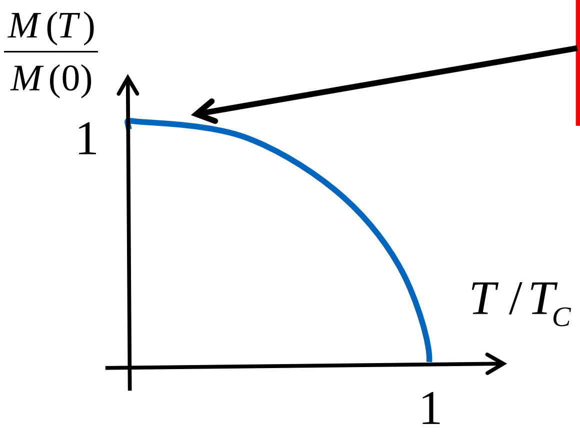
$\text{C-W law OK for } T >> \theta_{CW}$

$\chi_m \propto (T - \theta_{CW})^{-4/3} \text{ for } T \to \theta_{CW}$

Critical exponent (← spatial fluctuations?)

Spin Waves

(Elliott 7.2.5.4)



Mean field

$\sim 1\text{-}2\exp\left(-2\theta_{CW}/T\right)$

Excitations over Stoner gap

Experiment

$\sim 1\text{-}AT^{3/2}$

Spin waves as
low-energy excitations

Spin Waves

(Elliott 7.2.5.4)

Starting point, QM approach (Ascroft-Mermin pp. 701-703, 705)

1D Heisenberg model

Ladder operators

$$\hat{H}_{Heis} = -\sum_{i} \sum_{\delta}^{\text{nearest neighbors}} J_{i\delta} \hat{\mathbf{S}}_i \bullet \hat{\mathbf{S}}_{i\delta} = -\sum_{i\delta} J_{i\delta} \hat{S}_{z,i} \hat{S}_{z,i\delta} - \sum_{i\delta} J_{i\delta} \hat{S}_{-,i} \hat{S}_{+,i\delta}$$

[~AM(33.9)]

T = 0 Ground state [~AM(33.5)]

$$|0\rangle = \prod_i |\uparrow\rangle_i \; , \quad E = E_0, \quad \langle 0 | \hat{\mathbf{S}}^z_{tot} | 0 \rangle = NS$$

Site

••• ↑ ↑ ↑ ↑ ↑ ↑ ••• N spins (S)

Eigenstate of H_{Heis}

Stoner excitation [~AM(33.19)]

$$\left| \downarrow_j \right\rangle = |\downarrow\rangle_j \prod_{i \neq j} |\uparrow\rangle_i \quad ; \quad E = E_0 + 2\Delta$$

$$\langle \downarrow_j | \hat{\mathbf{S}}^z_{tot} | \downarrow_j \rangle = NS - 1$$

State

••• ↑ ↑ ↑ $\downarrow_j$ ↑ ↑ •••

Not an eigenstate, H ($S_{-,i}\, S_{+,i\delta}$) shifts excitation to the neighboring spin

Spin Waves

Spin wave (superposition)

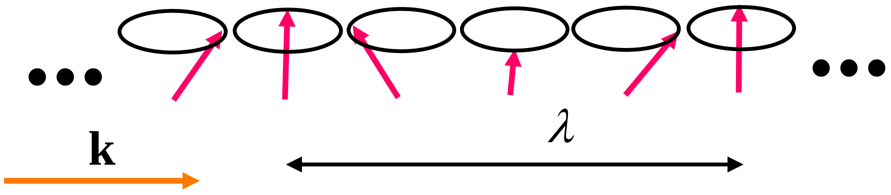
$$\left|\mathbf{k}\right\rangle = \sum_{j} \exp(i\mathbf{k}\bullet\mathbf{R}_j)\left|\downarrow_j\right\rangle$$

$$E = E(\mathbf{k}) \qquad \left\langle\mathbf{k}\right|\hat{\mathbf{S}}^z_{tot}\left|\mathbf{k}\right\rangle = NS-1$$

[~AM(33.23)]

Delocalized spin flip → $\left\langle\hat{S}^x_i\right\rangle = \left\langle\hat{S}^y_i\right\rangle \to 0$

Classically **S** precesses around *z* axis → Eigenstate



$\lambda \to \infty$

$\mathbf{k} \to 0$

$\mathbf{S}_i \parallel \mathbf{S}_{i+1}$

→

Low energy excitation

Change in exchange energy $\to 0$

$\Delta E(\mathbf{k}) \to 0$

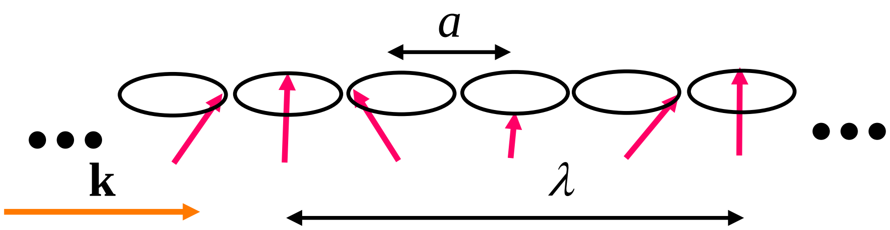
Classical solution of a 1D model (Elliott, Kittel)

QM - solution (Ashcroft - Mermin, Ibach - Lüth)

Spin Waves, Classical mechanics for a 1-dimensional model

➔ Dispersion relation *E* = *E*(**k**)

➔DOS(*E*) ➔*M*(*T*), (c_v…)



Total energy

$$E = -J\sum_{p}^{N}\mathbf{S}_p \bullet \mathbf{S}_{p+1}$$

Classical vectors

➔

p^{th} spin contribution

$$-J\mathbf{S}_p \bullet \left(\mathbf{S}_{p-1} + \mathbf{S}_{p+1}\right) = -\mathbf{m}_p \bullet \mathbf{B}_p$$

$$\mathbf{B}_p = \frac{J}{g_J \mu_B}\left(\mathbf{S}_{p-1} + \mathbf{S}_{p+1}\right)$$

[E(7.251,252)]

$$\mathbf{m}_p = g_J \mu_B \mathbf{S}_p$$

Time derivative of angular moment = torque

$$\frac{d(\hbar\mathbf{S}_p)}{dt} = \boldsymbol{\tau}_p = \mathbf{m}_p \times \mathbf{B}_p = \mathbf{S}_p \times J\left(\mathbf{S}_{p-1} + \mathbf{S}_{p+1}\right)$$

Equation of motion for interacting spins, nearest neighbor interactions only

[~E(7.255)]

Trial solution: propagating wave (cf. Classical lattice vibrations)

$$S_p^x = u\exp[i(kpa - \omega t)], \quad S_p^y = v\exp[i(kpa - \omega t)], \quad S_p^z = \text{constant}$$

[~E(7.259)]

Dispersion relation

➔ $$\hbar\omega = 2JS(1 - \cos(ka))$$

[E(7.260)]

Quadratic when *k* → 0

➔ $$\hbar\omega \approx JSa^2k^2$$

[E(7.261)]

No energy gap

Also in QM, see AM

Spin Waves, dispersion relation

(Elliott 7.2.5.4)

Dispersion relation

$$\boxed{\hbar\omega = 2JS(1-\cos(ka))}$$

[E(7.260)]

Also in QM, see AM

Quadratic when $k \rightarrow 0$

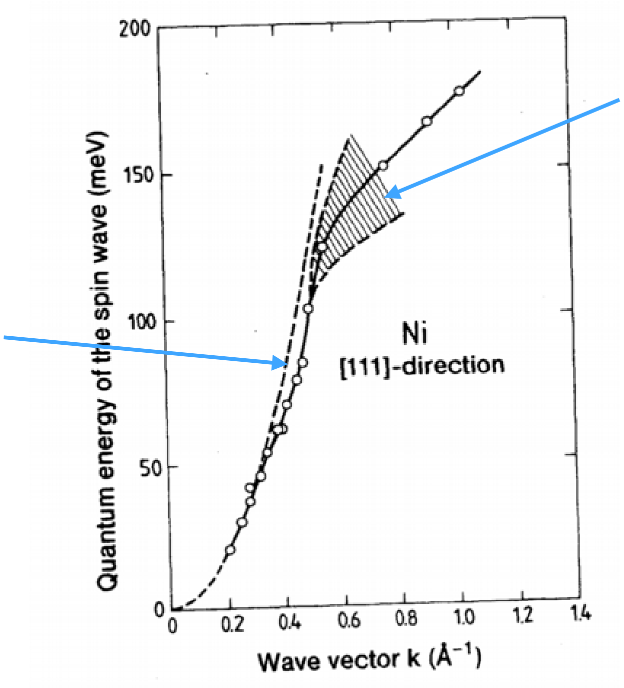
$$\rightarrow \boxed{\hbar\omega \approx JSa^2k^2}$$

[E(7.261)]

No energy gap

Experimental data for Ni

$\hbar\omega \propto k^2$



Stoner excitations

Finite spin-wave lifetimes

Magnons, quantized spin waves

cf. phonons

Total energy of magnons
with wave vector **k**

$$E_{\mathbf{k}} = (n+1/2)\hbar\omega_{\mathbf{k}}$$

[E(7.262)]

From dispersion relation

Number of excited magnons

Bose-Einstein distribution

$$n = \frac{1}{\exp\left(\hbar\omega_{\mathbf{k}} / k_B T\right) - 1}$$

Chemical potential = 0
➔ No limit for Σn

Total energy of all magnons

$$E_{tot} = \sum_{\mathbf{k}} (n+1/2)\hbar\omega_{\mathbf{k}}$$

Periodic boundary conditions

➡

N allowed **k** points in the 1st Brillouin zone.
N = number of unit cells in normalization volume *V*.

Magnons, temperature-dependent magnetization

Magnetization, $T = 0$

$$S_{tot}^{z} = NS$$

$$\boxed{M(0) = \frac{N}{V} g_J \mu_B S}$$

$T > 0$

Corresponding to each magnon

$$S_{tot}^{z} \rightarrow S_{tot}^{z} - 1$$

$$\boxed{M(T) = \frac{g_J \mu_B}{V}\left[NS - \sum_{\mathbf{k}} n(\mathbf{k},T) \right] = M(0)\left[1 - \frac{1}{NS} \sum_{\mathbf{k}} n(\mathbf{k},T) \right]}$$

[~E(7.263)]

$\mathrm{DOS} \propto \omega^{1/2}$ cf. 3D free electrons

$$\sum_{\mathbf{k}} n(\mathbf{k},T) \rightarrow \frac{V}{(2\pi)^3} \int d\mathbf{k} \frac{1}{\exp\left(\hbar\omega_{\mathbf{k}} / k_B T\right) - 1} \rightarrow \int d\omega \frac{g(\omega)}{\exp\left(\underbrace{\hbar\omega / k_B T}_{\rightarrow x}\right) - 1} \propto T^{3/2}$$

[~E(7.266)]

Bloch $T^{3/2}$ - law

$$\boxed{M(T) = M(0)\left(1 - \mathrm{constant} \times T^{3/2}\right)}$$

Magnons contribute also to specific heat (Exercise)

Lecture Assignment

Vining and Shelton, Phys. Rev. B **28**, 2732 (1983)

"A linear term in the heat capacity could be expected from two-dimensional ferromagnetic spin waves (with magnon dispersion relation proportional to q²) or from one-dimensional antiferromagnetic spin waves (with the magnetic dispersion relation proportional to q)."

Can we understand this statement?

$Gd_2Fe_3Si_5$

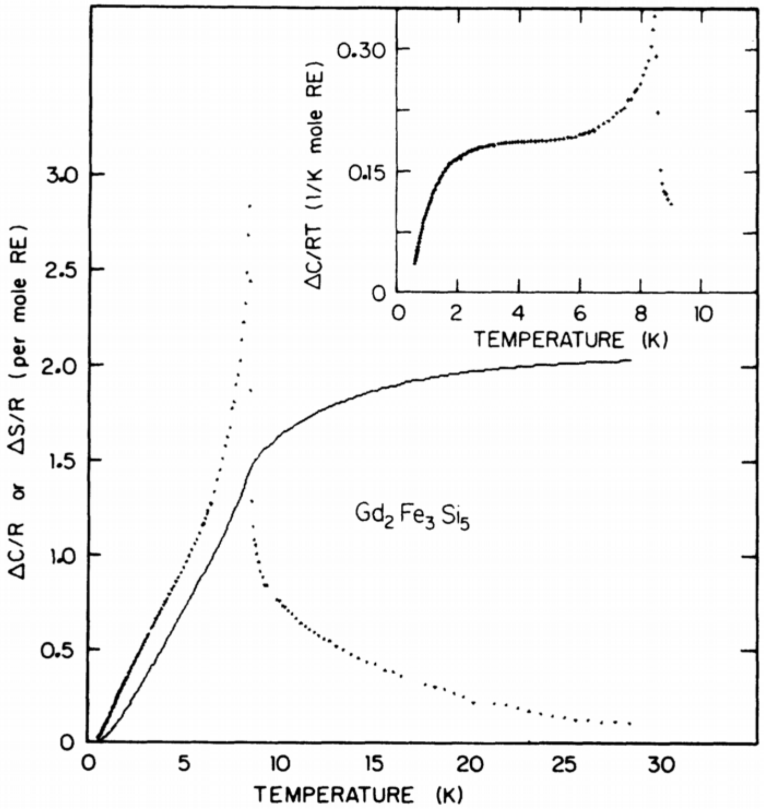


FIG. 2. Magnetic contribution to the heat capacity and entropy for $Gd_2Fe_3Si_5$. The inset indicates a large linear contribution to the heat capacity of $Gd_2Fe_3Si_5$ between 2 and 6 K.

Lecture Assignment

Bosonic systems

$$C_v = \frac{dE}{VdT} \propto \frac{d}{dT}\int_{-\infty}^{+\infty} E g(E) n(E,T) dE \quad ; \quad \boxed{g(E)\propto E^n} \quad ; \quad n(E) = \frac{1}{\exp\underbrace{\left(E/k_BT\right)}_{x}-1}$$

$$x = E/k_BT \quad \Rightarrow \quad C_v \propto \frac{d}{dT}\int_{-\infty}^{+\infty}(k_BTx)^{n+2}\frac{1}{\exp(x)-1}dx \quad \Rightarrow \quad \boxed{C_v\propto T^{n+1}}$$

Density of states *g*(*E*) 🡐 Dimension *d*, dispersion relation *E*=*E*(**k**):

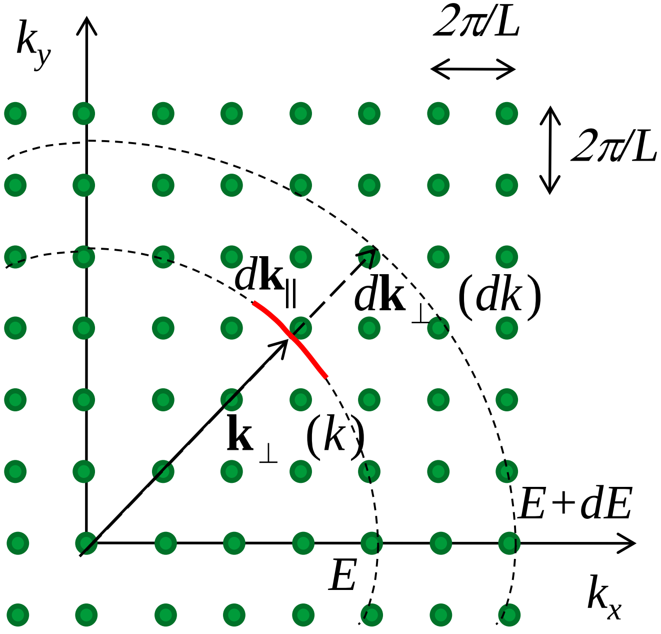
Periodic boundary conditions of a "cube" L^d

$$\mathbf{k}\text{ -space } g(\mathbf{k}) = \frac{L^d}{(2\pi)^d}, \quad \text{constant}$$

$$\mathrm{g}(E)dE = \left[\mathbf{k}-\text{values }\underbrace{\text{within } dE\ldots E+dE}_{}\right]/L^d = g(\mathbf{k})\Delta\mathbf{k}$$

Isotropic dispersion, $E(\mathbf{k}) = E(k)$:

$$\mathrm{g}(E)dE = g(\mathbf{k})dk_{(\perp)}\int d\mathbf{k}_{\parallel} \quad \Rightarrow \quad \boxed{\mathrm{g}(E)\propto\frac{dk}{dE}\int d\mathbf{k}_{\parallel}}$$



Lecture Assignment

Density of states *g*(*E*) 🡰 Dimension *d*, dispersion relation *E*=*E*(**k**), Big picture

	$E = E(\mathbf{k})$	free electrons magnons $E \propto k^2$	Debye model acoustic phonos $E \propto k$
	dk / dE		
d	$\int d\mathbf{k}_{\parallel}$	$g(E) \propto E^n$	$g(E) \propto E^n$
3			
2			
1			

FILL IN THE EMPTY BOXES; CORRECT ANSWERS ON THE LAST SLIDE

Lecture Assignment

Bosonic systems

$$\boxed{g(E) \propto E^n} \quad \Rightarrow \quad \boxed{C_v \propto T^{n+1}}$$

Temperature dependencies:
- -3D acoustic phonon specific heat at low temperatures ~T^3
- -3D magnon specific heat at low temperatures ~$T^{3/2}$
- -2D ferromagnetic magnon specific heat at low temperatures ~T
- -1D antiferromagnetic magnon specific heat at low temperatures ~T

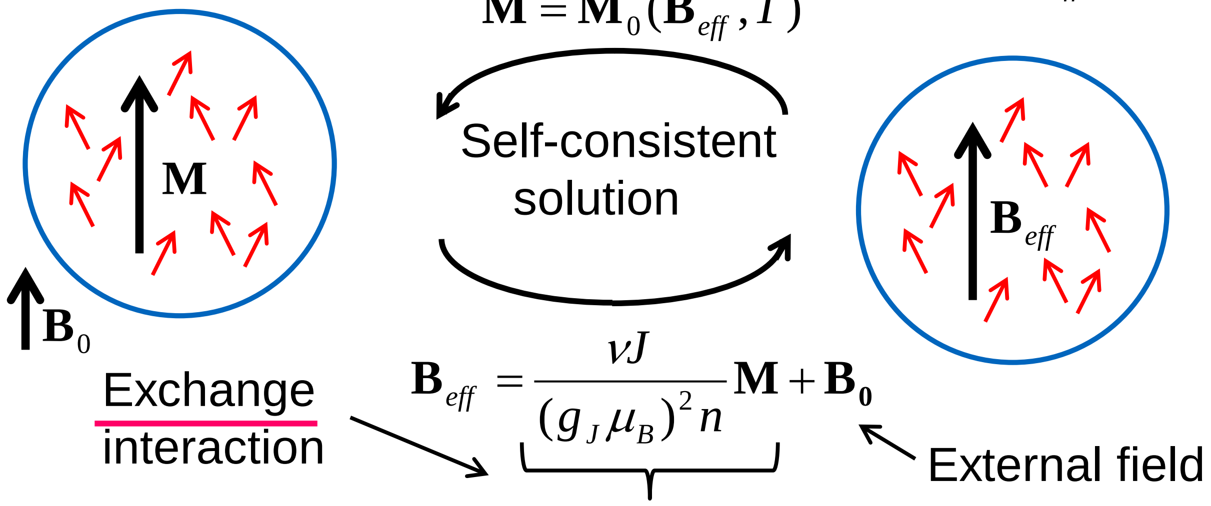
-Magnon influence on magnetization at low temperatures ~(1 - constant $T^{3/2}$)

-Thermal conductivity $\kappa = l_{mfp} v c_v / 3$

-3D electron (Fermions!) specific heat at low temperatures ~T

Ferromagnetism of Localized Moments, Summary

Mean-Field Theory



Ordered system of paramagnetic ions with magnetization **M**

Noninteracting gas of paramagnetic ions in field $\mathbf{B}_{eff}$

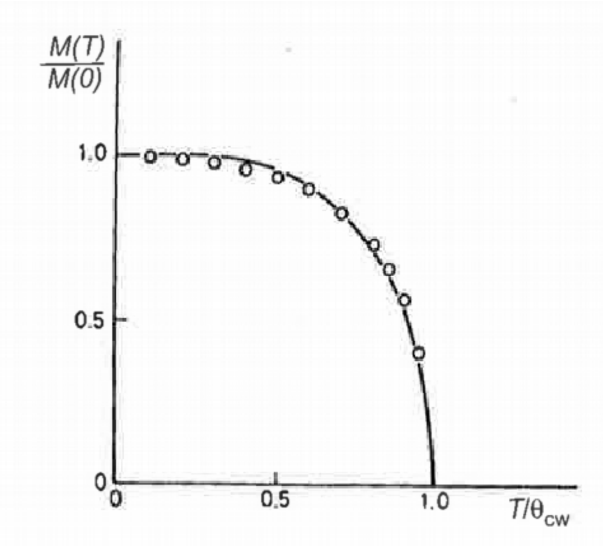
$$\mathbf{M} = \mathbf{M}_0(\mathbf{B}_{eff}, T)$$

Self-consistent solution

$$\mathbf{B}_{eff} = \underbrace{\frac{\nu J}{(g_J \mu_B)^2 n}}\mathbf{M} + \mathbf{B_0}$$

Exchange interaction

External field



$\theta_{CW} = \nu J / 4k_B$

Curie - Weiss temperature

$B_0 = 0:$

$T < \theta_{CW} \Rightarrow$ Ferromagnetism, $M > 0$

$T > \theta_{CW} \Rightarrow$ Paramagnetism, $M = 0$

M in different temperature regions:

$T << \theta_{CW}$: $M / M_{\max} \approx 1 - 2\exp\left(-2\theta_{CW} / T\right)$

$T \approx< \theta_{CW}$: $M / M_{\max} \approx \sqrt{3}\left(1 - T/\theta_{CW}\right)^{1/2}$

$T > \theta_{CW}$: $\chi = \dfrac{C}{T - \theta_{CW}}$ Curie - Weiss law

Ferromagnetism of Localized Moments, Summary

Beyond the Mean-Field Theory, magnons

Mean field theory:

Excitations over the Stoner gap

$\Rightarrow M / M_{\max} \approx 1 - 2\exp\left(-2\theta_{CW} / T\right)$

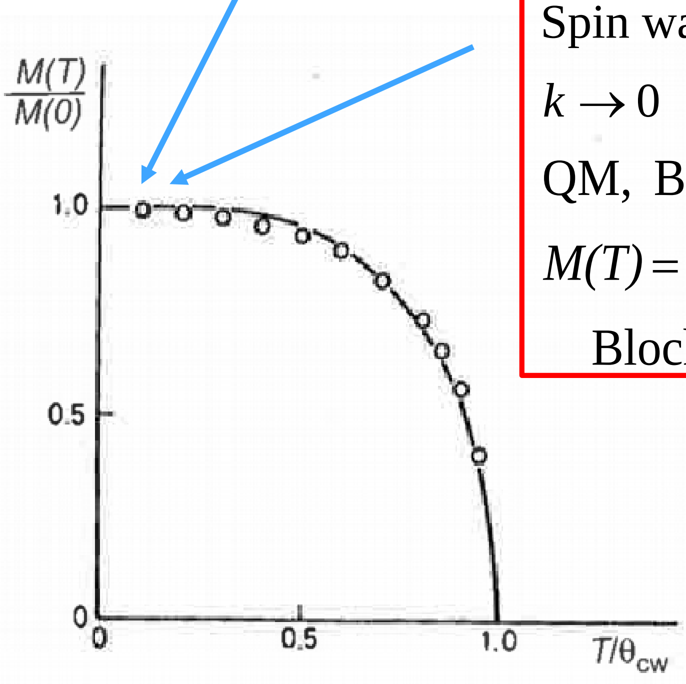
Spin waves

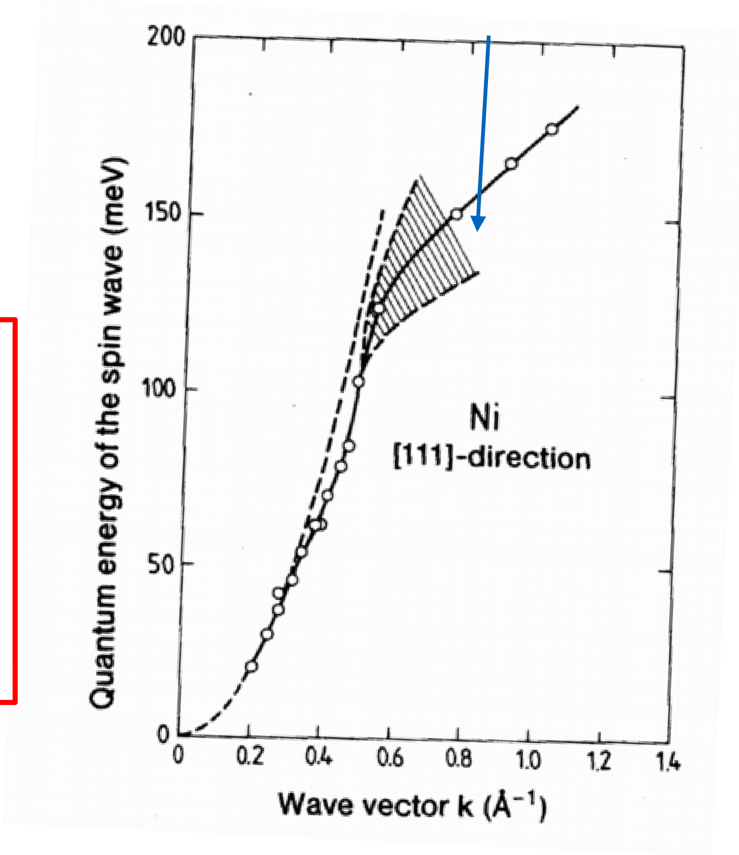
$k \to 0 \quad \Rightarrow \quad \omega \propto k^2$, no gap!

QM, Bose-Einstein statistics

$M(T) = M(0)\left(1 - \text{constant} \times T^{3/2}\right)$

Bloch $T^{3/2}$-law





Magnetic properties

- Response of materials to an external magnetic field
  - Magnetic quantities, magnetism is quantum mechanics (home work)
  - Quantum mechanical description
  - Atomic diamagnetism, paramagnetism (lecture work)
  - Response of free electron gas

- Spontaneous magnetism (Ferromagnetism and antiferromagnetism)
  - Exchange interaction, H_2 molecule, free electron gas
  - Mean-field approximation for ferromagnetism of magnetic moments
  - Spin waves (low-energy excitations)
  - Stoner model for ferromagnetism of itinerant electrons
  - Antiferromagnetism
  - Domain structure

The last lecture

Lecture Assignment

Density of states $g(E)$ 🡐 Dimension d, dispersion relation $E=E(\mathbf{k})$, Big picture

	$E = E(\mathbf{k})$	free electrons magnons $E \propto k^2$	Debye model acoustic phonos $E \propto k$
	dk/dE	$\propto 1/k$	Constant
d	$\int d\mathbf{k}_{\parallel}$	$g(E) \propto E^n$	$g(E) \propto E^n$
3	spherical surface $\propto k^2$	$\propto k$ $\propto \sqrt{E}$	$\propto k^2$ $\propto E^2$
2	circle circumference $\propto k$	Constant MOSFET 2D ferr. magnons?	$\propto k$ $\propto E$ Dirac Fermions in graphene
1	line section Constant	$\propto 1/k$ $\propto 1/\sqrt{E}$ Landau tubes carbon nanotubes	Constant 1D antiferr. magnons?