A Hybrid Genetic Algorithm for the Interaction of Electricity Retailers with Demand Response

Maria João Alves[1(✉)], Carlos Henggeler Antunes[2], and Pedro Carrasqueira[3]

[1] Faculty of Economics, University of Coimbra/INESC Coimbra, Coimbra, Portugal
mjalves@fe.uc.pt

[2] Department of Electrical Engineering and Computers, University of Coimbra/INESC Coimbra, Coimbra, Portugal
ch@deec.uc.pt

[3] INESC Coimbra, Coimbra, Portugal
pmcarrasqueira@net.sapo.pt

Abstract. In this paper a bilevel programming model is proposed for modeling the interaction between electricity retailers and consumers endowed with energy management systems capable of providing demand response to variable prices. The model intends to determine the optimal pricing scheme to be established by the retailer (upper level decision maker) and the optimal load schedule adopted by the consumer (lower level decision maker) under this price setting. The lower level optimization problem is formulated as a mixed-integer linear programming (MILP) problem. A hybrid approach consisting of a genetic algorithm and an exact MILP solver is proposed. The individuals of the population represent the retailer's choices (electricity prices). For each price setting, the exact optimal solution to the consumer's problem is obtained in a very efficient way using the MILP solver. An illustrative case is analyzed and discussed.

Keywords: Genetic algorithm · Bilevel problem · Mixed-integer linear programming · Demand response · Electricity retail market

1 Introduction

The retail electricity market has been mostly working as a one-way communication scheme. The retailer buys the energy in the wholesale market at variable prices, which depend on the purchase time and the demand profile. As the consumers are, in general, charged at a flat rate, they are indifferent to price oscillations and lack the stimulus to engage in distinct consumption patterns according to their flexibility of use of appliances. If consumers could see prices changing along the day, i.e. they were offered dynamic tariffs within some contracted bounds, they would expectedly adopt actions, namely by means of automated energy management systems, to schedule their loads to minimize the electricity bill without jeopardizing the quality of the energy services provided by the appliances, namely comfort requirements. Profiting from the flexibility consumers have in scheduling load operation would be of utmost importance to

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several players in the electricity industry chain. Consumers could see their electricity bill decreasing and retailers could make more judicious decisions regarding buying and selling electricity. Also network companies, both at distribution and transmission levels, would benefit because they could use demand-side resources to mitigate congestion and make a better management of the availability of distributed generation based on renewable sources.

In this setting, the retailer could reflect the acquisition conditions onto the consumers, e.g. by determining variable electricity prices, which is not feasible in the traditional one-side electricity market. Retailers and consumers have conflicting goals. Retailers want to maximize profits by selling electricity subject to the regulation framework, and consumers want to minimize costs subject to requirements of quality of the energy services associated with the operation of loads in appropriate time slots. In addition to conflicting goals, there is a hierarchical relation between retailers and consumers as the former determine prices and the latter react by scheduling their loads accordingly. This is a bilevel optimization problem.

Communication capabilities associated with the evolution of the electricity system to smart grids lay the foundations for bi-directional interaction between retailers and consumers. This enables that time-varying price information is sent by the retailer, which in turn receives the response of the consumer by adjusting the operation schedule of the loads with the aim to minimize the electricity bill.

Several models have been proposed in literature concerning demand side management (DSM) in the residential setting [1]. DSM has many beneficial effects, enabling a better usage of available generation capacity and network infrastructures, contributing to avoid or postpone new investments, decreasing peak load demand, reducing the carbon emission levels and improving the overall grid sustainability. DSM programs have re-emerged in the smart grid context allowing end-users reshaping their energy consumption pattern and taking advantage of dynamic tariffs.

Several approaches have been proposed using time-varying pricing strategies to decrease peak load. In the day-ahead hourly pricing strategy, the consumers receive the next 24 h prices a day or some hours before. Consumers should then react accordingly by scheduling their appliances to get a satisfactory trade-off between minimizing the electricity bill and maximizing or imposing constraints on their welfare regarding comfort requirements.

In this paper a bilevel programming model is proposed for modeling the interaction between the electricity retailer and consumers. Bilevel models for this purpose have been studied in the literature. Zugno et al. [2] considered a theoretical game to establish a Stackelberg relationship between retailers and consumers in a dynamic pricing framework, with a stochastic component. In Bu et al. [3] consumers aim at maximizing utility, which derives from the amount of energy consumed, the price and an individual factor associated with each consumer. The retailer aims at maximizing the profit considering that it can buy energy from two suppliers with distinct prices and degree of certainty. Yang et al. [4] incorporated DSM through an interaction game between retailers and consumers according to their utility functions. The consumer's objective function derives from energy cost and the utility of energy consumption, which depends on the difference between the actual consumption and a target. Zhang et al. [5] presented a bilevel model with multiple objective functions in the upper level, which aim at maximizing the profit of supply companies, and a single objective in the lower level, which aims at minimizing the consumer's electricity bill. The consumer can choose the supply company.

Hybrid approaches have been proposed by Meng and Zeng [6,7] to solve bilevel problems with one leader (retailer), who wants to maximize profit, and multiple consumers, who want to minimize their bills. Meng and Zeng [6] considers interruptible, non-interruptible and curtailable appliances, which lead to three lower level separate sub-optimization problems. In addition to consumers using demand optimization, [7] also considers costumers whose energy consumption patterns are not known to the retailer. Therefore, these patterns should be learned by the retailer with the purpose of retail price determination. Both approaches [6,7] use genetic algorithms to solve the profit maximization problem at the retailer's side and an LP solver to derive optimal solutions at the consumers' side.

The bilevel model proposed in this paper intends to determine the optimal pricing scheme to be established by the retailer and the optimal load schedule adopted by the consumer under this price setting. Consumers are able to deviate consumption of shiftable loads, i.e. cyclic loads as dishwashers, laundry etc., to lower price periods subject to time slot constraints for load operation, which can decrease the retailer's profits. The structure of the paper is as follows. In Sect. 2 the main concepts of bilevel models are outlined. In Sect. 3 new bilevel formulations for modeling the interaction between the retailer and consumer optimization problems are presented. In Sect. 4 an algorithmic approach combining a genetic algorithm (GA) with a mixed-integer linear programming (MILP) exact solver is described. Numerical results and the ensuing discussion are presented in Sect. 5. In Sect. 6 the main conclusions are drawn.

2 Bilevel Programming

In bilevel optimization problems the upper level decision maker (*leader*) controls decision variables x, while the lower level decision maker (*follower*) controls decision variables y. The two decision makers have their own objective functions, which are subject to interdependent constraints. The decision process is sequential as the leader makes his decisions first by setting the values of x. Then, the follower reacts by choosing the y values that optimize his objective function on the feasible solutions restricted by the fixed x. The goal of the leader is to optimize his objective function, but he must incorporate into the optimization process the reaction of the follower because it affects the leader's objective value.

The general bilevel programming problem can be stated as follows (BP):

$$\begin{aligned}
&\min_{x\in X} F(x,y)\\
&s.t.\ G(x,y) \le 0\\
&\qquad y \in \arg\min_{y\in Y} \{f(x,y) : g(x,y) \le 0\}
\end{aligned}$$

where $X \subset \mathbb{R}^{n_1}$ (n_1 being the number of upper level variables) and $Y \subset \mathbb{R}^{n_2}$ (n_2 being the number of lower level variables) are closed sets. $F(x, y)$ and $f(x, y)$ are the leader's and the follower's objective functions, respectively. Since the follower optimizes $f(x, y)$ after x has been selected, x is a constant vector whenever $f(x, y)$ is optimized. For fixed $x \in X$, the set $Y(x) = \{y \in Y : g(x, y) \leq 0\}$ is the *feasible set* of the follower. The set $\Psi(x) = \left\{ y \in Y : y \in \underset{y' \in Y(x)}{\arg\min} f(x, y') \right\}$ is called the follower's *rational reaction set* to a given x. The feasible set of (BP), also called the *induced region*, is $IR = \{(x, y) : x \in X, G(x, y) \leq 0, y \in \Psi(x)\}$. It is difficult to find global optimal solutions to bilevel optimization problems due to their inherent non-convexity. Even the linear bilevel problem is NP-hard [8].

3 Bilevel Formulations for the Interaction Between the Retailer's and Consumer's Optimization Problems

We consider a bilevel problem to model the interaction between the retailer and consumers. The retailer buys energy in the wholesale market and sells it to consumers. The retailer wants to maximize profit while consumers aim at minimizing the cost of their energy consumption. In this model a partially flexible consumer is considered, who can decide on the allocation of some shiftable loads based on a price schedule communicated by the retailer. Shiftable loads are typically cyclic loads, such as dishwashers or laundry machines, whose operation cycle can be shifted in time but not interrupted. The model considers a cluster (aggregation) of consumers with similar consumption and demand response profiles, thereafter referred to as the consumer.

The problem has a bilevel structure, where the retailer (*leader*) determines the prices x_i to be charged to the consumer (*follower*) in each predefined sub-period P_i ($i = 1, \cdots, I$) of the planning period T. Thus, the number of upper level variables is I, i.e., the number of sub-periods P_i. As proposed by Zugno et al. [2], in order to enforce market competitiveness of retailer prices, we introduce constraints on x_i imposing minimum ($\underline{x}_i$) and maximum ($\overline{x}_i$) values in each sub-period P_i and an average price (x^{AVG}) value during T.

Knowing the electricity prices, the consumer determines the time (z_j) each flexible load $j \in \{1, \ldots, J\}$ must start to minimize the cost of electricity and ensuring that the operation cycle of load j is within a specified *comfort time slot* $T_j = [T1_j, T2_j] \subseteq$ T.

Data:

T = number of intervals (minutes, quarter-hour, half-hour or other period of time) of the planning period ($t = 1, \cdots, T$). Let T= $\{1, \cdots, T\}$.

J = number of shiftable loads to be managed by the consumer ($j = 1, \cdots, J$).

I = number of sub-periods of time $P_i \subset$ T in which different prices of electricity (time-of-use tariffs) are charged by the retailer to the consumer ($i = 1, \cdots, I$).

$P1_i$, $P2_i$: points in time that delimit each sub-period P_i, $i = 1, \cdots, I$, such that $P_i = [P1_i, P2_i]$ and $\bigcup_{i=1}^{I} P_i$ =T. Let $\bar{P}_i$ denote the amplitude of P_i, i.e. $\bar{P}_i = P2_i - P1_i + 1$.

$\underline{x}_i$ = minimum price charged to the consumer in sub-period P_i.
$\bar{x}_i$ = maximum price charged to the consumer in sub-period P_i.
x^{AVG} = average price charged to the consumer in T.
π_t = energy price seen by the retailer in the spot market at time $t \in$ T (€/KWh $\times(m/60)$ where m is the number of minutes in one unit of time t).
C_t = contracted power by the consumer at time t of the planning period (KW).
b_t = non-controllable base load at time t of the planning period (KW), i.e. amount of load that cannot be scheduled by the consumer's energy management system.
d_j = duration of the operation cycle of shiftable load j.
$f_j(r)$ = power requested by load j at time r of its operation cycle ($r = 1, \cdots, d_j$) (KW).
$Tj = [T1_j, T2_j] \subseteq$ T: time slot in which load j is allowed to operate.
Upper level decision variables:
x_i = price charged by the retailer to the consumer during sub-period P_i (€/KWh $\times(m/60)$ where m has the same meaning as above), $i = 1, \cdots, I$.
Lower level decision variables:
z_j = starting time of the operation cycle of load j, $j = 1, \cdots, J$.
Auxiliary lower level variables:
u_{jt} = binary variable representing whether the operation cycle of load j is "on" or "off" at time t of the planning period, $j = 1, \cdots, J$, $t = 1, \cdots, T$.
p_{jt} = power requested to the grid by load j at time t of the planning period (KW), $j = 1, \cdots, J$, $t = 1, \cdots, T$.

Bilevel Model 1.

$$max\ F = \sum_{i=1}^{I} \sum_{t \in P_i} x_i(b_t + \sum_{j=1}^{J} p_{jt}) - \sum_{t=1}^{T} \pi_t(b_t + \sum_{j=1}^{J} p_{jt}) \tag{1}$$

$s.t.$

$$x_i \leq \bar{x}_i\ , i = 1, \cdots, I \tag{2}$$

$$x_i \geq \underline{x}_i\ , i = 1, \cdots, I \tag{3}$$

$$\frac{1}{T} \sum_{i=1}^{I} \bar{P}_i x_i = x^{AVG} \tag{4}$$

$$min\ f = \sum_{i=1}^{I} \sum_{t \in P_i} x_i(b_t + \sum_{j=1}^{J} p_{jt}) \tag{5}$$

$s.t.$

$$u_{jt} = \begin{cases} 1 \text{ if } z_j \leq t \leq z_j + d_j \\ 0 \quad\quad \text{otherwise} \end{cases}, \quad j = 1, \cdots, J; t = 1, \cdots, T \tag{6}$$

$$p_{jt} = f_j(t - z_j + 1)u_{jt}, \qquad j = 1, \cdots, J; t = 1, \cdots, T \tag{7}$$

$$\sum_{j=1}^{J} p_{jt} + b_t \leq C_t, \qquad t = 1, \cdots, T \tag{8}$$

$$T1_j \leq z_j \leq T2_j - d_j + 1 \qquad j = 1, \cdots, J \tag{9}$$

The objective function at the upper level (1) is the maximization of the retailer's profit (revenue from selling energy to consumer minus cost of purchasing the energy in the spot market). Constraints (2) to (4) define the limits for the energy prices charged to the consumer in each sub-period P_i and set an average price in T.

The formulation of the lower level problem in Model 1 is based on the DSM model proposed in [1]. The objective function (5) consists in the minimization of the consumer's total cost; (6) sets the value of the auxiliary binary variables u_{jt} as function of the variables z_j and time t; variables u_{jt} are, in turn, used in equations (7), which set the value of the power requested to the grid by each load j at each time t according to the load operation cycle; constraints (8) impose that the contracted power is not exceeded at any time and constraints (9) impose time limits for the operation of each load according to the time slots defined by the consumer.

Model 1 can be written in an equivalent manner by reformulating the lower level problem as a MILP problem. Thus, bilevel Model 2 is presented below, which is a mathematical programming model equivalent to bilevel Model 1. It does not consider variables z_j, $j = 1, \cdots, J$ (which specify the starting time t of the operation cycle of load j), but rather binary variables w_{jrt} that indicate whether the load j is "on" or "off" at time t of the planning period and it is at time r of its operation cycle. Explicit variables u_{jt} are no longer necessary because they can be expressed in terms of w_{jrt}: $u_{jt} = \sum_{r=1}^{d_j} w_{jrt}$. The upper level variables (x_i) are the same as in Model 1.

Data:

The data are the same as in Model 1 with a single difference: the functions $f_j(r)$ are replaced by series of discrete values, consisting of one f_{jr} value for each combination j, r. Thus:

f_{jr} = power requested by load j at time r of its operation cycle ($r = 1, \cdots, d_j$) (KW).

To avoid ambiguity between points in time of the operation cycle and points in time of the planning period, we refer to the "time r" of the operation cycle as "stage r".

Lower level decision variables:

w_{jrt} = binary variable representing whether load j is "on" or "off" at time t of the planning period and at stage r of its operation cycle.

In order not to unnecessarily increase the number of w_{jrt} variables, they are defined only for t in the time slot allowed for the operation for each load. Therefore, w_{jrt} are defined for $j = 1, \cdots, J$, $r = 1, \cdots, d_j$, $t = T1_j, \cdots, T2_j$.

Auxiliary lower level variables:

p_{jt} = power requested to the grid by load j at time t of the planning period (KW), $j = 1, \cdots, J$, $t = 1, \cdots, T$.

Bilevel Model 2.

$$max\ F = \sum_{i=1}^{I}\sum_{t\in P_i} x_i(b_t + \sum_{j=1}^{J} p_{jt}) - \sum_{t=1}^{T}\pi_t(b_t + \sum_{j=1}^{J} p_{jt}) \tag{10}$$

$s.t.$

$$x_i \leq \bar{x}_i\ , i = 1, \cdots, I \tag{11}$$

$$x_i \geq \underline{x}_i\ , i = 1, \cdots, I \tag{12}$$

$$\frac{1}{T}\sum_{i=1}^{I} \bar{P}_i x_i = x^{AVG} \tag{13}$$

$$min\ f = \sum_{i=1}^{I}\sum_{t\in P_i} x_i(b_t + \sum_{j=1}^{J} p_{jt}) \tag{14}$$

$s.t.$

$$p_{jt} = \sum_{r=1}^{d_j} f_{jr} w_{jrt}, \qquad j = 1, \cdots, J; t = T1_j, \cdots, T2_j \tag{15}$$

$$p_{jt} = 0, \qquad j = 1, \cdots, J; t < T1_j \vee t > T2_j \tag{16}$$

$$\sum_{j=1}^{J} p_{jt} + b_t \leq C_t, \qquad t = 1, \cdots, T \tag{17}$$

$$\sum_{r=1}^{d_j} w_{jrt} \leq 1, \qquad j = 1, \cdots, J; t = T1_j, \cdots, T2_j \tag{18}$$

$$w_{j(r+1)(t+1)} \geq w_{jrt}, \quad j = 1, \cdots, J; r = 1, \cdots, d_j - 1; t = T1_j, \cdots, T2_j - 1 \tag{19}$$

$$\sum_{t=T1_j}^{T2_j} w_{jrt} = 1, \qquad j = 1, \cdots, J; r = 1, \cdots, d_j \tag{20}$$

$$\sum_{t=T1_j}^{T2_j - d_j + 1} w_{j1t} \geq 1, \qquad j = 1, \cdots, J \tag{21}$$

$$w_{jrt} \in \{0, 1\}\,, \quad j = 1, \cdots, J; r = 1, \cdots, d_j; t = T1_j, \cdots, T2_j$$
$$p_{jt} \geq 0 \qquad j = 1, \cdots, J; t = 1, \cdots, T \tag{22}$$

The upper-level problem (10)–(13) and the lower-level objective function (14) are the same as in Model 1.

Constraints (15)–(16) correspond to (7) in Model 1 and aim at setting the auxiliary variables p_{jt}. Since these variables are defined for every $t = 1, \cdots, T$, these constraints comprise two groups: (15) which define p_{jt} for t within the time slot allowed for load j to operate (for which w_{jrt} variables have been defined) and (16) for t outside this time slot in which p_{jt} is always zero.

Constraints (17) are the same as (8) in Model 1.

Constraints (18) ensure that, at time t of the planning period, each load j is either "off" or is "on" at only one stage r of its operation cycle.

Constraints (19) ensure that, for each load j, if it is "on" at time t and at stage $r \leq d_j - 1$ of its operation cycle, then it must be also "on" at time $t+1$ and at stage $r+1$.

Constraints (20) ensure each load j is operating at stage r exactly once. Note that constraints (19) do not prevent that a load j starts at a time after $T2_j - d_j + 1$ and, as it cannot finishes until $T2_j$, it continues from $T1_j$. For instance, consider that a load j is at stage $r = 1$ at $t = T2_j - 1$, $r = 2$ at $t = T2_j$ and then skips to $r = 3$ at $t = 1$, $r = 4$ at $t = 2$, etc.; this operation scheme is not feasible in practice but it does not violate constraints (19). Thus, constraints (21) are imposed, which ensure that each load j starts its operation (stage $r = 1$) at most at time $T2_j - d_j + 1$ so that it can finish not later than $T2_j$, i.e. within its allowed comfort time slot. Constraints (19) together with (20) and (21) ensure that load j is operating exactly d_j consecutive time intervals, forcing w_{jrt} to be 0 when load j is "off".

4 A Hybrid Genetic Algorithm with MILP Solver

A hybrid approach consisting of a GA and an exact MILP solver is proposed to solve the bilevel programming problem formulated in Model 2 for the interaction between the electricity retailer and the consumer.

The GA applies to the upper level problem (10)–(13). Each individual of the population represents an electricity price setting $x' = (x'_1, x'_2, \cdots, x'_I)$. For each x' the lower level problem (14)–(22) with $x = x'$ is exactly solved. Let y' be the optimal solution obtained for this lower level instance (note that the lower level decision variables are w_{jrt} and p_{jt}). Each solution (x', y') to the bilevel problem is then evaluated by the upper level objective function F in (10). Hence, the fitness function is $F(x, y)$.

The lower level problem has been modeled using the AMPL language [9] and the GA has been coded in Delphi for Windows. For each individual x', the lower level MILP problem is exactly solved by the CPLEX solver called from the GA. The electricity prices (x_i) are the only parameters that change from one call to another one. The general description of the GA is presented below.

Genetic Algorithm:

Create the initial population *Pop* of N individuals $x' = (x'_1, x'_2, \cdots, x'_I)$ satisfying constraints (11)–(13), as described in Sect. 4.1.
For each individual x' in *Pop*, solve the lower level problem (14)–(22) with $x = x'$ using the MILP solver. Let y' be the optimal solution obtained.
Evaluate the fitness of each solution (x', y') to the bilevel problem by calculating $F(x', y')$ according to (10).
while the stopping condition is not met **do**
  repeat
    Select two parents x' and x'' from *Pop* and apply crossover to generate a child x^c.

Apply mutation to x^c with probability P_m.

Repair x^c to satisfy constraints (11)–(13) or discard it if it is not repairable.

until N children have been generated, which form the set *Offspring* (see Sect. 4.2).

For each x^c in *Offspring* solve the lower level problem (14)–(22) with $x = x^c$ using the MILP solver. Let y^c be the optimal solution obtained.

Evaluate the fitness of each solution (x^c, y^c) by calculating $F(x^c, y^c)$ according to (10).

Create *NextPop* by copying the best solution obtained thus far (which is either in *Pop* or in *Offspring*) and performing $N-1$ binary tournaments without replacement between individuals of *Offspring* and *Pop*. Update the current population *Pop* with *NextPop*.

end while

return (x', y') of *Pop* with the highest fitness.

4.1 Initial Population

The initial population consists of N individuals $x' = (x'_1, x'_2, \cdots, x'_I)$ in which each x'_i is randomly generated in the range $[\underline{x}_i, \bar{x}_i]$. In order to ensure that x' also satisfies the average price constraint (13), the following repair procedure is applied.

Repair x':

Compute $s = \sum_{i=1}^{I} \bar{P}_i x'_i$

Let A be the set of indices i of x'_i that are allowed to be changed. Initially, $A = \{1, 2, \cdots, I\}$.

if $s \neq Tx^{AVG}$ **then**

  Let $\Delta = Tx^{AVG} - s$

  Let $P = \sum_{i \in A} \bar{P}_i$

  for each $x'_i, i \in A$ **do**

    $x'_i \leftarrow x'_i + \Delta/P$

  end for

end if

for $i = 1$ to I **do**

  if $x'_i < \underline{x}'_i$ **then**

    $x'_i \leftarrow \underline{x}'_i$ and $A \leftarrow A \setminus \{i\}$

  else if $x'_i > \bar{x}'_i$ **then**

    $x'_i \leftarrow \bar{x}'_i$ and $A \leftarrow A \setminus \{i\}$

  end if

end for

Compute $s = \sum_{i=1}^{I} \bar{P}_i x'_i$

if $s = Tx^{AVG}$ **then**

  Stop and return x'

else if $A = \oslash$ **then**

  Stop and discard x'

21: **else**
22: **go to** 2:
23: **end if**

The process of randomly generating x' and repairing it using the above procedure is repeated until N individuals are generated. In our experiments the repair procedure converged in few iterations.

4.2 Reproduction Process

The reproduction process creates N offspring, each one generated from a different selection of two parents in the current population *Pop*. A binary tournament is applied in which two individuals from *Pop* are chosen at random and the best one is selected to be one parent. In the crossover process there is a 50 % of chance of this individual being the first or the second parent. The other parent is randomly selected from *Pop*. A one-point crossover operator is then applied to produce an offspring from the two selected parents. Hence, if the first parent is $x' = (x'_1, x'_2, \cdots, x'_I)$ and the second one is $x'' = (x''_1, x''_2, \cdots, x''_I)$, the offspring is $x^c = (x'_1, \cdots, x'_{i_1}, x''_{i_1+1}, \cdots, x''_I)$ where i_1 is the crossover point drawn at random between 2 and $I - 1$.

Mutation is then applied to x^c with a probability P_m of changing each gene of x^c. For a given x^c_i, the mutation consists of adding or subtracting a positive perturbation randomly generated in the range between 0 and $0.2(\bar{x}_i - \underline{x}_i)$. If x^c_i is out of the bounds imposed by constraints (11) and (12), then it is pushed to the closest bound and its index is excluded from the set A of variables that are allowed to be changed in the repair procedure. x^c is then repaired (using the procedure described in Sect. 4.1) to satisfy constraint (13) or is discarded if it is not repairable. This process is repeated until N offspring have been generated. They form the *Offspring* population, which will compete with *Pop* to determine the population for the next generation.

The individual with the best fitness obtained thus far always survives from one generation to the next (i.e. an elite set with one element is considered). This is the first individual inserted into the next population. The other $N - 1$ individuals are selected by binary tournament selection between an individual from the *Offspring* population and an individual from the current population *Pop* (the parents), both chosen at random. The individual with the highest fitness wins and is selected to integrate the next population. Any individual included in the new population is removed from its original population (*Offspring* or *Pop*), so the same individual cannot be selected twice.

5 Numerical Results and Discussion

An illustrative case is discussed in this section. Most data were obtained from actual audit information and some values were estimated. A 24 h planning period divided into intervals of 15 min is considered. Thus, 1 unit of time (t) is a quarter-hour, which leads to a planning period of $T = 96$ units of time, T=$\{1, \cdots, 96\}$.

Since original data have been collected for 1 min periods, those values were aggregated (by considering average values) for the quarter-hour intervals.

Five shiftable loads were considered ($J = 5$): dishwasher, laundry machine, electric water heater (EWH), electric vehicle and clothes dryer. Figure 1 shows the operation cycles of these loads, i.e. f_{jr} values. The time slots $[T1_j, T2_j]$ allowed for the different loads j are displayed in Fig. 2.

Seven sub-periods of time $P_i \subset$ T, $i = 1, \cdots, 7$, were considered for defining the electricity prices to be charged by the retailer to the consumer. The maximum and minimum prices ($\bar{x}_i$ and $\underline{x}_i$) in each sub-period are displayed in Fig. 3. The last time point $P2_i$ that delimits each sub-period P_i , $i = 1, \cdots, I$, is also presented in Fig. 3 below the curve of the minimum price. The first time point $P1_i$ is always given by $P2_{i-1} + 1$, being 1 for $i = 1$.

The energy prices the retailer has to pay (π_t) for the electricity bought in the spot market are displayed in Fig. 4. All prices in Figs. 3 and 4 are in €/KWh, so they were then converted to periods of quarter-hours (i.e. divided by 4) to feed the model. The average price $x^{AVG} = 0.116$ €/KWh was considered.

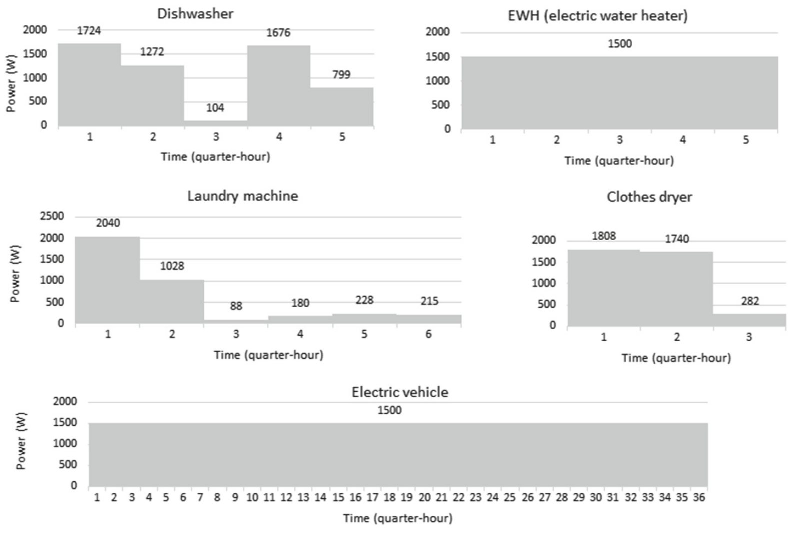

Fig. 1. Operation cycles of the loads to be managed.

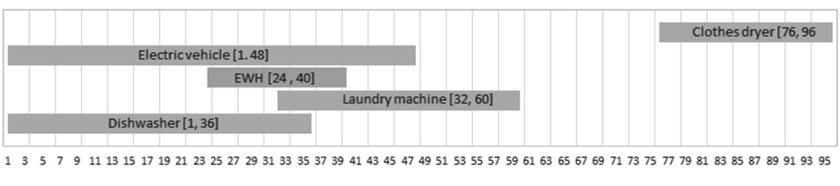

Fig. 2. Comfort time slots allowed for the operation of each load.

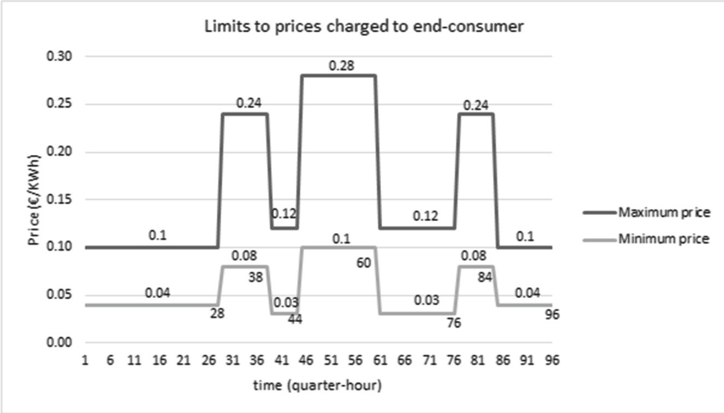

Fig. 3. Minimum and maximum electricity prices charged to the consumer.

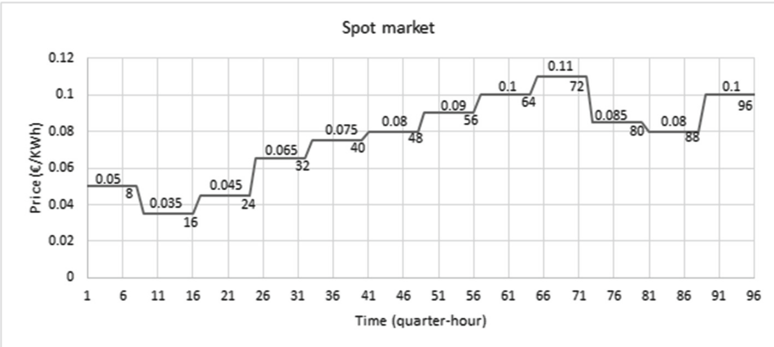

Fig. 4. Prices at spot market.

The diagram of non-controllable base load ($b_t, t = 1, \cdots, T$) is presented in Fig. 5. The contracted power C_t is 4.6 KW for $t = 28, \cdots, 84$ and 3 KW for the other $t \in$T.

In the computational simulations a population size of 30 individuals was considered and 100 iterations of the GA were performed in each run.

We started by tuning the parameters and two values for the probability of mutation were tested: $P_m = 0.05$ and $P_m = 0.01$. Five runs for each P_m were performed and systematically the run with $P_m = 0.05$ yielded a final solution better than with $P_m = 0.01$. Each run with $P_m = 0.01$ considered equal seed for the generation of random numbers as the corresponding run with $P_m = 0.05$. Figure 6 illustrates the evolution of the best and average values of F among the population members of each generation in one run with $P_m = 0.05$ and the corresponding run with $P_m = 0.01$. In general, the algorithm with $P_m = 0.01$ converges quickly for solutions with higher fitness F but afterwards it has difficulty in improving them significantly. Thus, we have adopted $P_m = 0.05$.

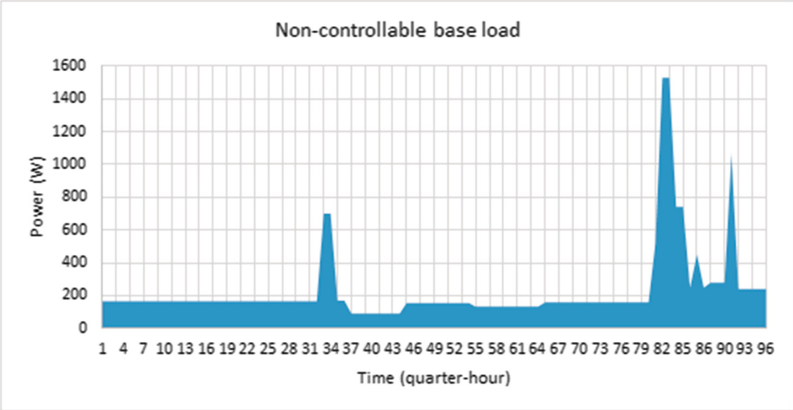

Fig. 5. Power requested to the grid by the base load.

Analyzing the solutions computed over the generations, we could observe that the schedule plans of the shiftable loads (optimal solutions to the lower level problem for each setting of the upper level variables) are few and there are not many differences among them. However, there are very significant differences in the retailer's profit (F) and the follower's cost (f) among the solutions due to the variation of the electricity prices (values of the upper level variables). Remind that the bilevel problem is the retailer's problem, so the optimal solution is the one that offers the retailer the highest profit knowing that the consumer will schedule loads at his best convenience considering the retailer's price setting.

For each upper level variable configuration given by the GA, the lower level is exactly solved. In the present case, this is a MILP problem with 2230 binary variables (w_{jrt}), 151 continuous variables (p_{jt}, excepting those that are necessarily 0 due to constraints (16)), 2233 inequality constraints and 206 equality constraints. Despite being a large problem it is solved very fast by CPLEX as the optimal solution was always found at the root of the branch-and-bound tree and only a few simplex iterations (less than 20) were needed to solve each instance. So, the exact resolution of the lower level problem has revealed to be a very interesting option for this model. The simulations were done in a computer with an Intel Core i7-2600K CPU@3.4 GHz and 8 GB RAM. On average, the time spent in performing one complete generation with a population of 30 individuals (i.e. steps 5: to 12: of the Genetic Algorithm, including solving 30 lower level problems) was 7 s.

Twenty independent runs with 100 iterations each (considering $P_m = 0.05$) were performed and the best solution in each run was recorded (solution with highest fitness, F^{best}). Let us call this set of solutions the *20-best set*. The maximum, minimum, mean and standard deviation of F in the *20-best set* are reported in Table 1. Among these best solutions there are only two distinct schedule plans for the shiftable loads. Plan 1 considers the following starting times for the loads: dishwasher - $t = 1$; laundry machine - $t = 41$; EWH - $t = 36$;

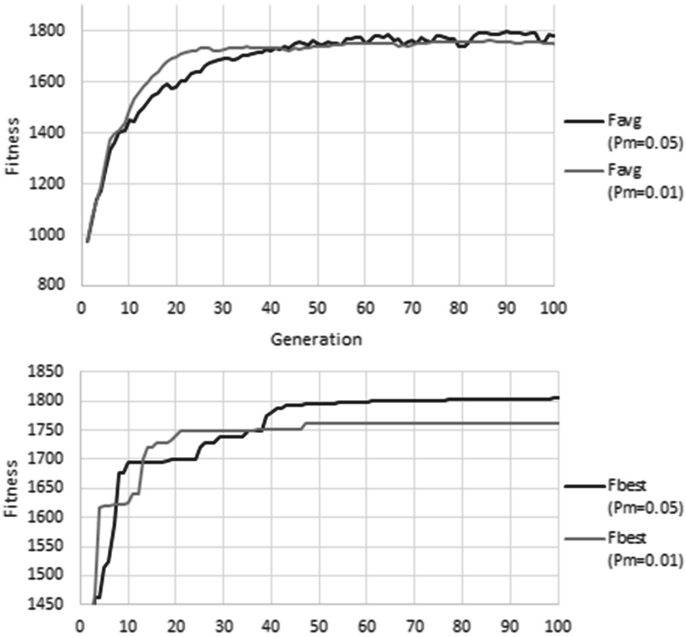

Fig. 6. Evolution of F^{avg} and F^{best} over 100 generations for $P_m = 0.05$ and $P_m = 0.01$.

Table 1. Statistics of F^{best} (€) in 20 independent runs.

maximum F^{best}	minimum F^{best}	mean F^{best}	standard deviation of F^{best}
1825.66	1770.68	1807.43	14.38

electric vehicle - $t = 5$; clothes dryer - $t = 85$. Plan 2 only differs from Plan 1 in the time operation of the laundry machine. The starting times in Plan 2 are the following: dishwasher - $t = 1$; laundry machine - $t = 48$; EWH - $t = 36$; electric vehicle - $t = 5$; clothes dryer - $t = 85$.

The solution with maximum F^{best} in the *20-best set* presents a retailer's profit (F) of 1825.66 and the consumer's cost (f) is 3368.24; the electricity prices in each of the 7 sub-periods of time are: $x_1 = 0.1$, $x_2 = 0.23998$, $x_3 = 0.119964$, $x_4 = 0.106572$, $x_5 = 0.0301812$, $x_6 = 0.235864$, $x_7 = 0.095124$ €/KWh. The schedule plan is Plan 2 whose load diagram is depicted in Fig. 7. The minimum F^{best} is 1770.68 and the consumer's cost in this solution is 3308.91. It corresponds to schedule Plan 1. All monetary values (F and f) are in € and refer to a period of 24 h and a cluster of 1000 consumers with similar consumption and demand response profiles.

We made further experiments with a higher number of iterations (1000). These experiments led to small improvements (0.37 % better than the best solution described above) in the upper level solution and similar plans for the lower level solutions but with a significantly heavier computation burden.

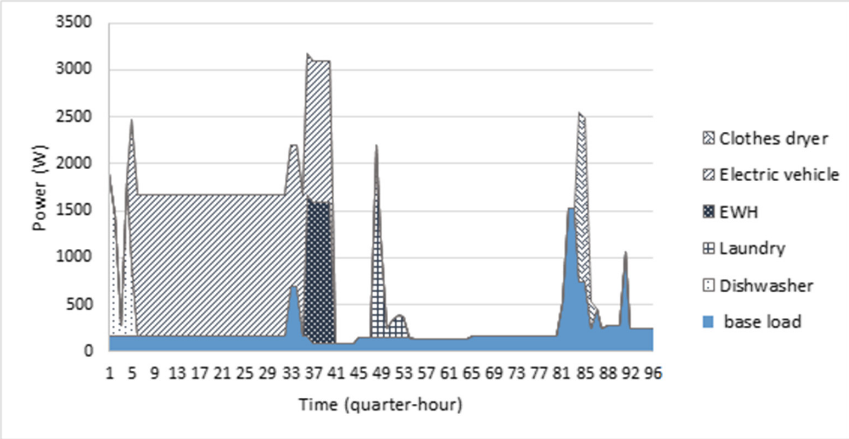

Fig. 7. Load diagram corresponding to the best solution.

6 Conclusions

In this paper new bilevel formulations for modeling the interaction between the retailer and consumers in the electricity market were presented. A bilevel problem is a programming problem where a (lower level) optimization problem is embedded as a constraint in another (upper level) optimization problem. In the present model, the electricity retailer is the upper level decision maker, which buys energy in the spot market and sells it to consumers. The retailer determines prices to be charged to the consumer, subject to the regulation framework, with the aim of maximizing its profit. The consumer (lower level decision maker) reacts to the prices by scheduling loads to minimize his electricity bill.

Bilevel programming problems are very difficult to solve due to their inherent non-convexity. In this paper we proposed a genetic algorithm combined with an exact MILP solver to tackle the problem.

An illustrative case was studied considering real data for the loads, which were obtained through audits. The exact resolution of the lower level problem for each upper level setting (electricity prices) has revealed to be a very interesting option, thus leading to a very efficient hybrid approach combining a GA with a MILP solver. Although the lower level problem is a high dimensional MILP problem, it is solved very fast.

As future work we intend to include other type of loads, in particular thermostatic controlled loads. We also plan to expand the model to consider multiple objective functions at the lower level to analyze consumer's cost vs. comfort trade-offs. For this purpose, time slot constraints for the load operation can be replaced by preferred time slots and the deviation from those slots (as measure of discomfort) is minimized.

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