Urban Economics

Lecture 2: Monocentric city model

Spring 2024
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Monocentric city model

In this lecture, we analyze the monocentric city model

Origins in the work of Alonso (1964), Mills (1967), and Muth (1969)

- Alonso, W. (1964): *Location and land use*. Cambridge: Harvard University Press.
- Mills, E. (1967): An Aggregative Model of Resource Allocation in a Metropolitan Area. *American Economic Review* 57(2), 197–210.
- Muth, R. (1969): *Cities and housing*. Chicago: University of Chicago Press.

Monocentric city model

Main goal of the model is to explain the empirical regularities that we observe in real-life cities

Main mechanism is the relationship between commuting costs, housing prices, and housing consumption

Other key ingredients are identical utility levels and developer profits across space, i.e. spatial equilibrium within the city

Outline

Empirical regularities of real-life cities

Monocentric city model assumptions

Consumer analysis

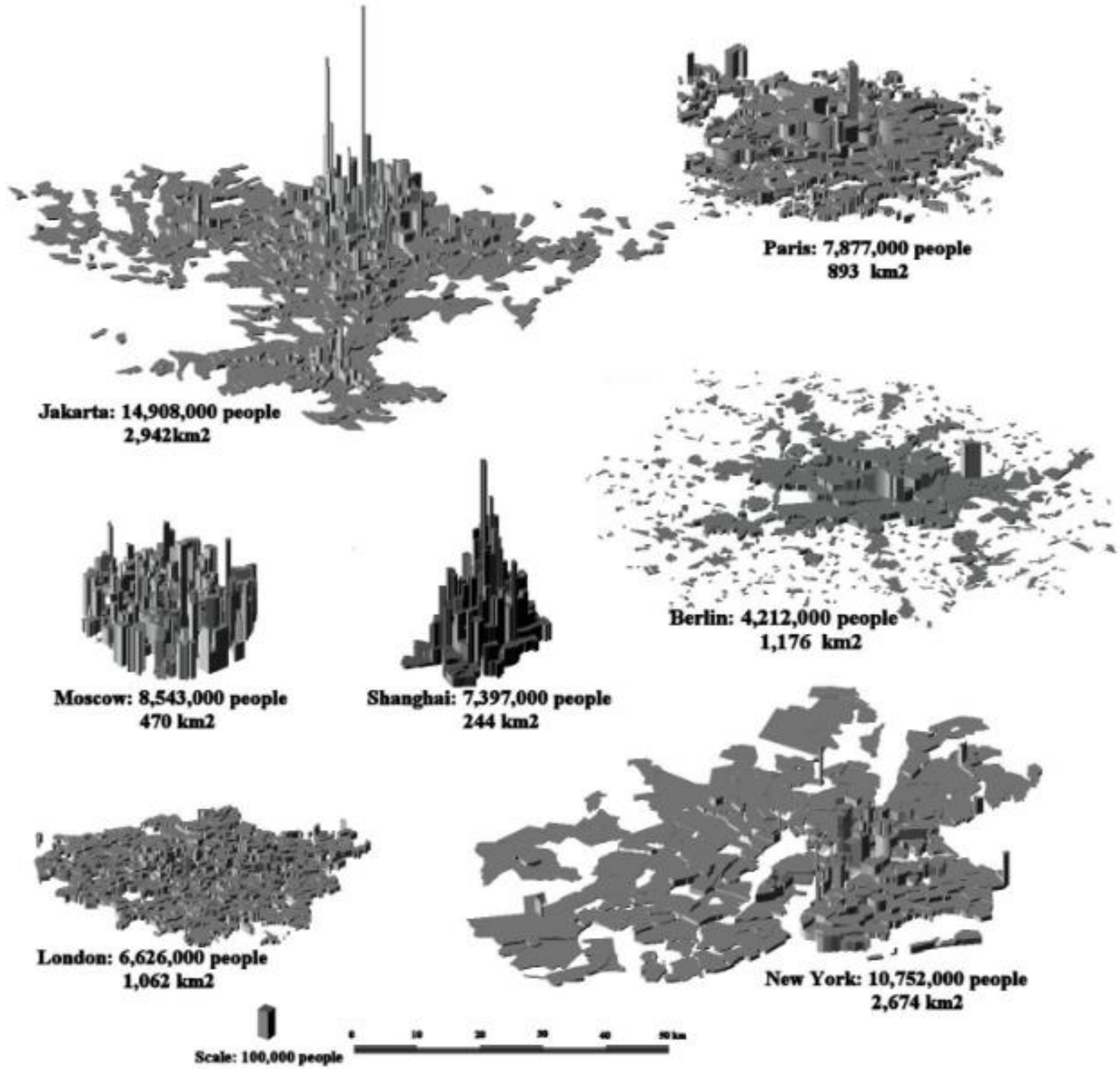
Producer analysis

Empirical example

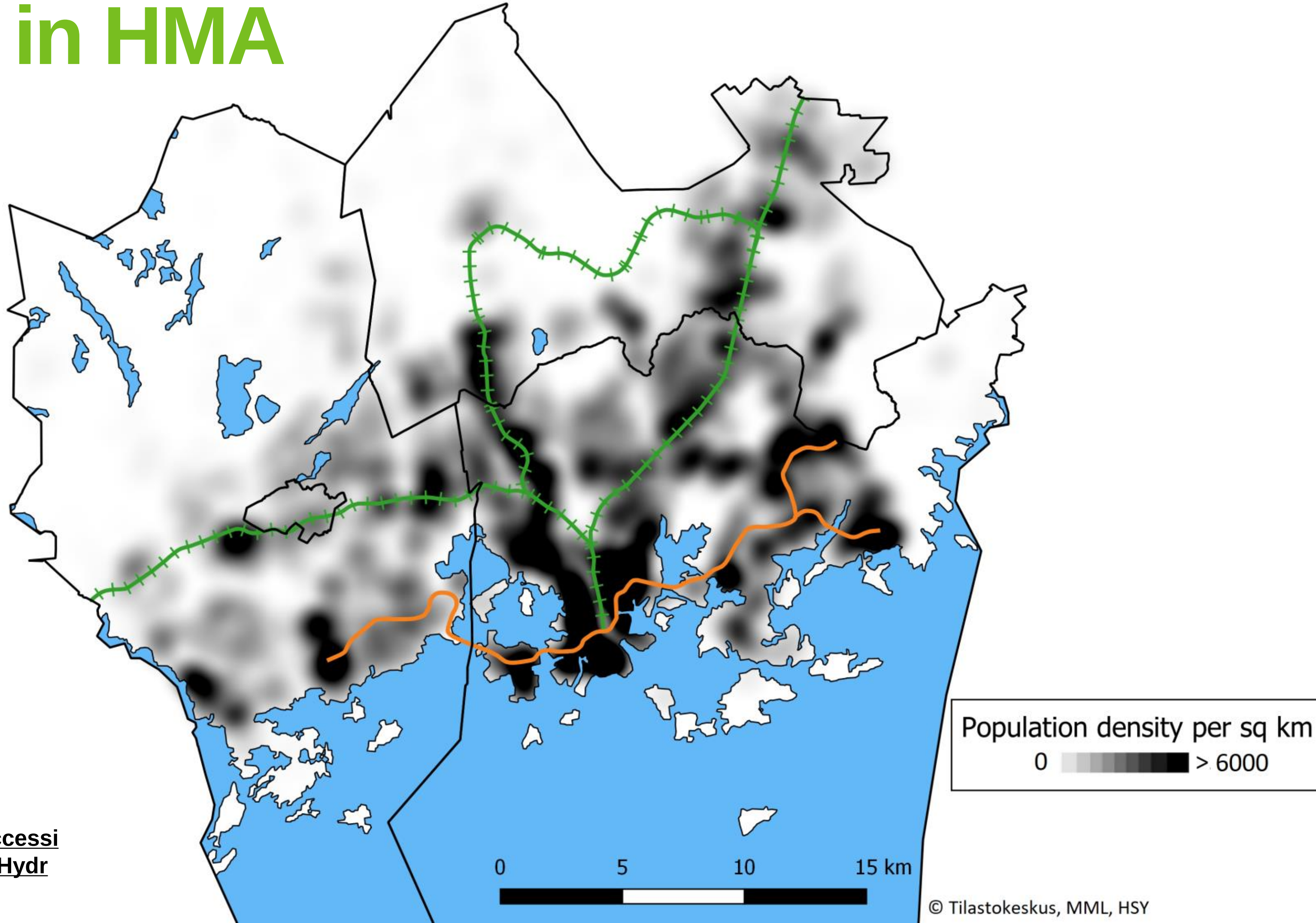
This lecture follows Brueckner’s Chapter 2

Stylized facts about inner structure of cities

Population density in 7 major cities

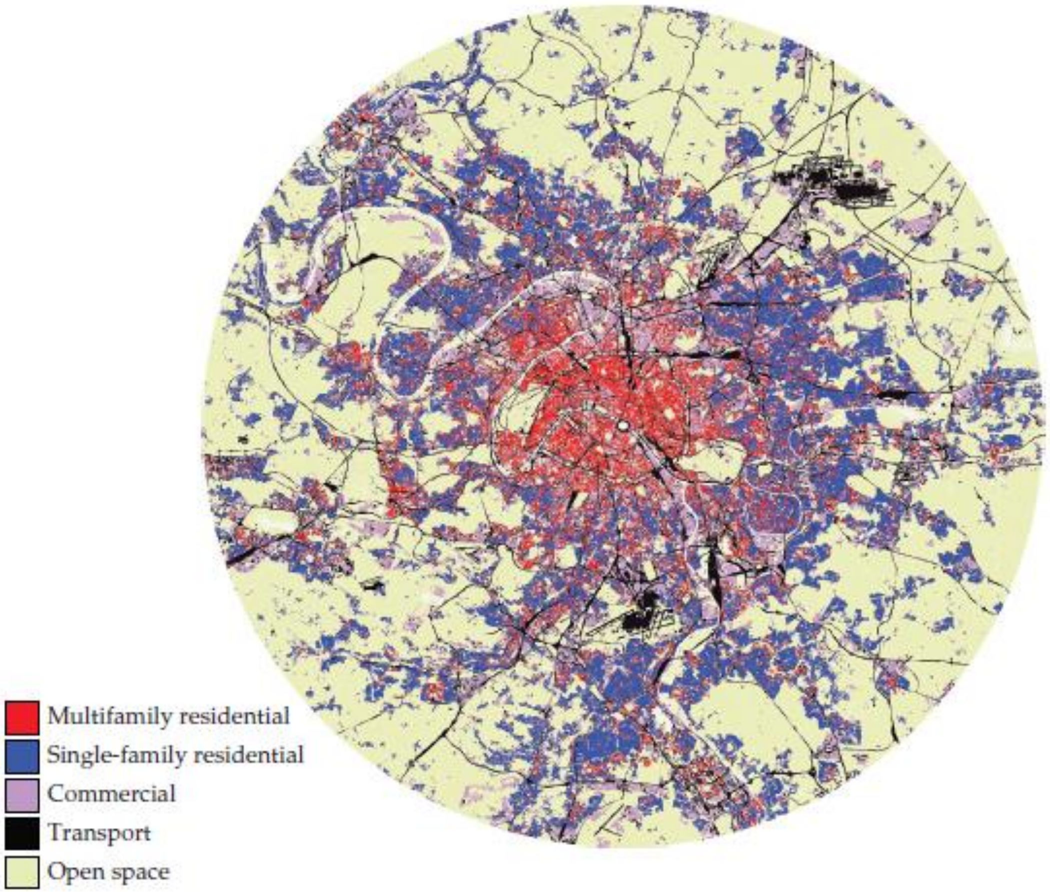


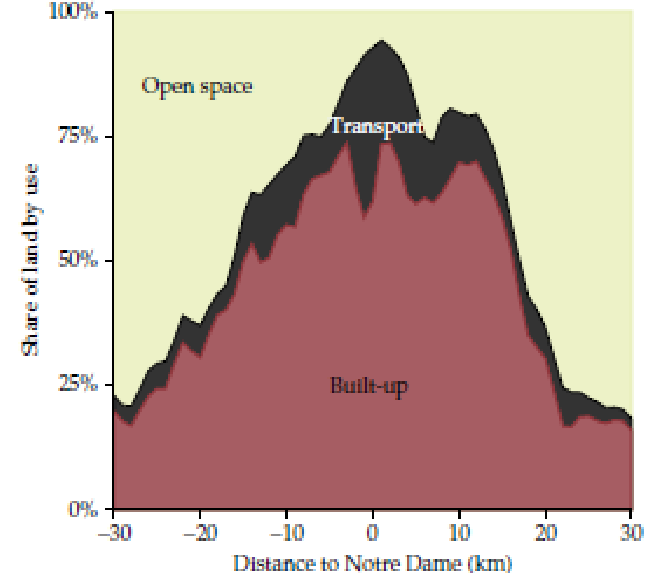
Population density in HMA

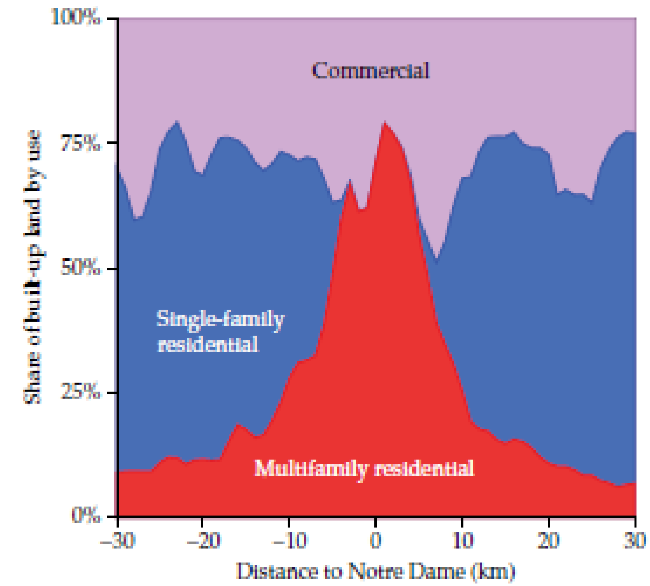


https://blogs.helsinki.fi/accessibility/files/2018/10/HSPA_Hydro_pulse50ms.gif

Land use in Paris

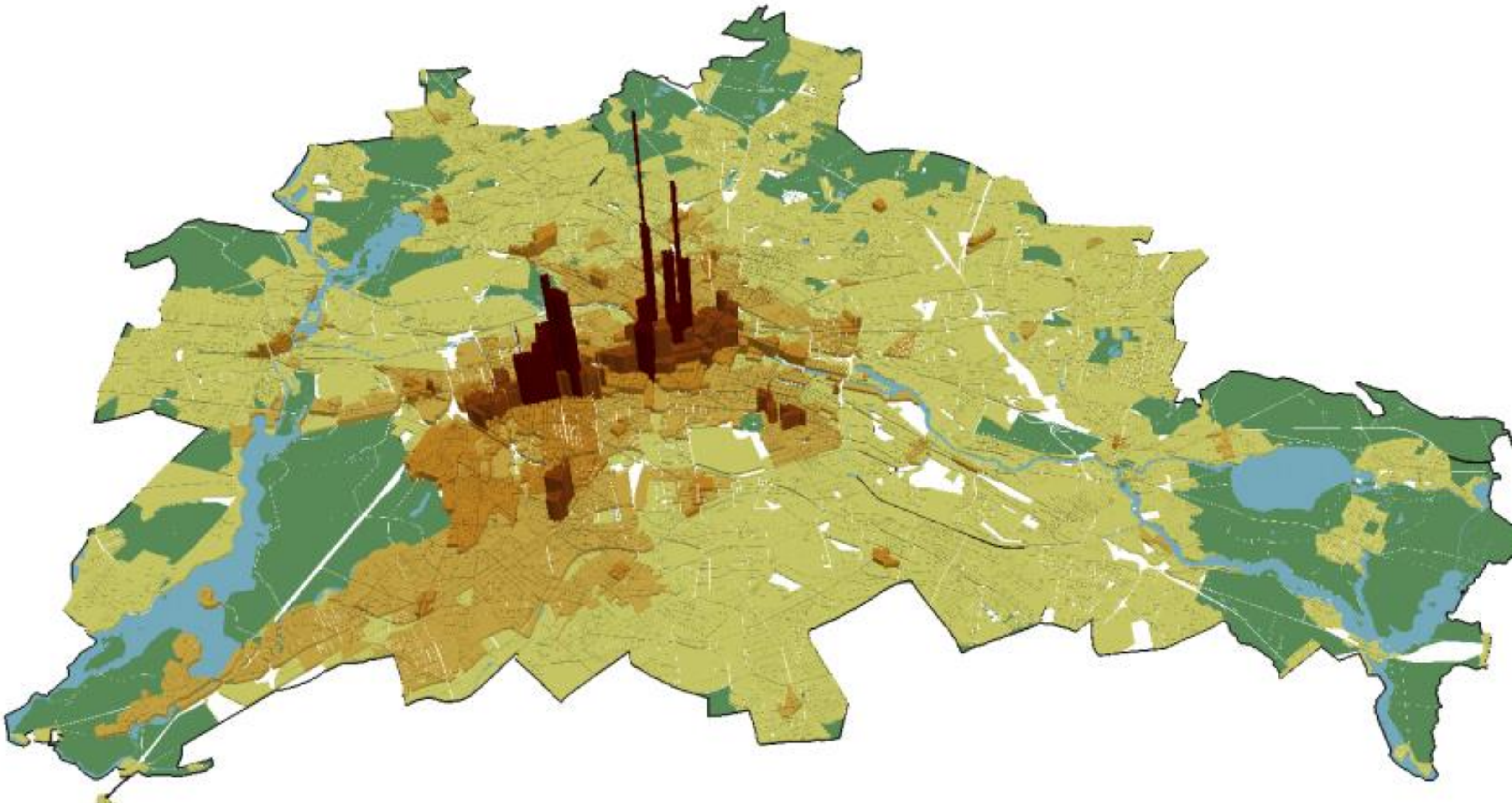






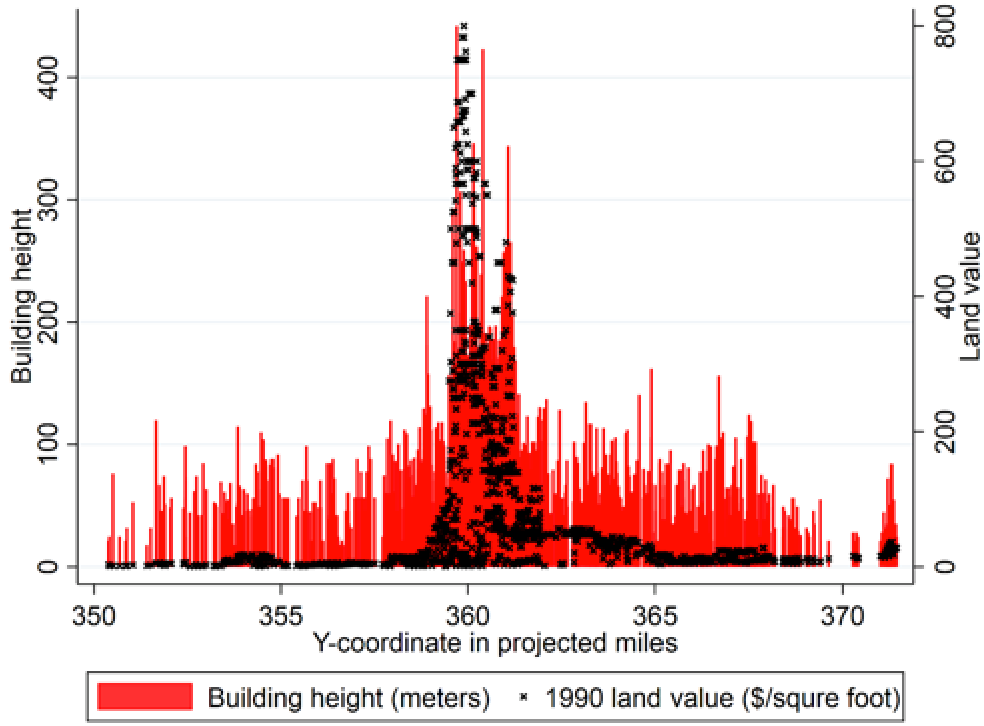
Duranton, G. & D. Puga. 2015. Urban Land Use. In G. Duranton, J.V. Henderson, W.C. Strange (ed.), *Handbook of Regional and Urban Economics*, Vol 5, 467-560

Land prices in Berlin



Source: Ahlfeldt et al. (2015), *Econometrica*

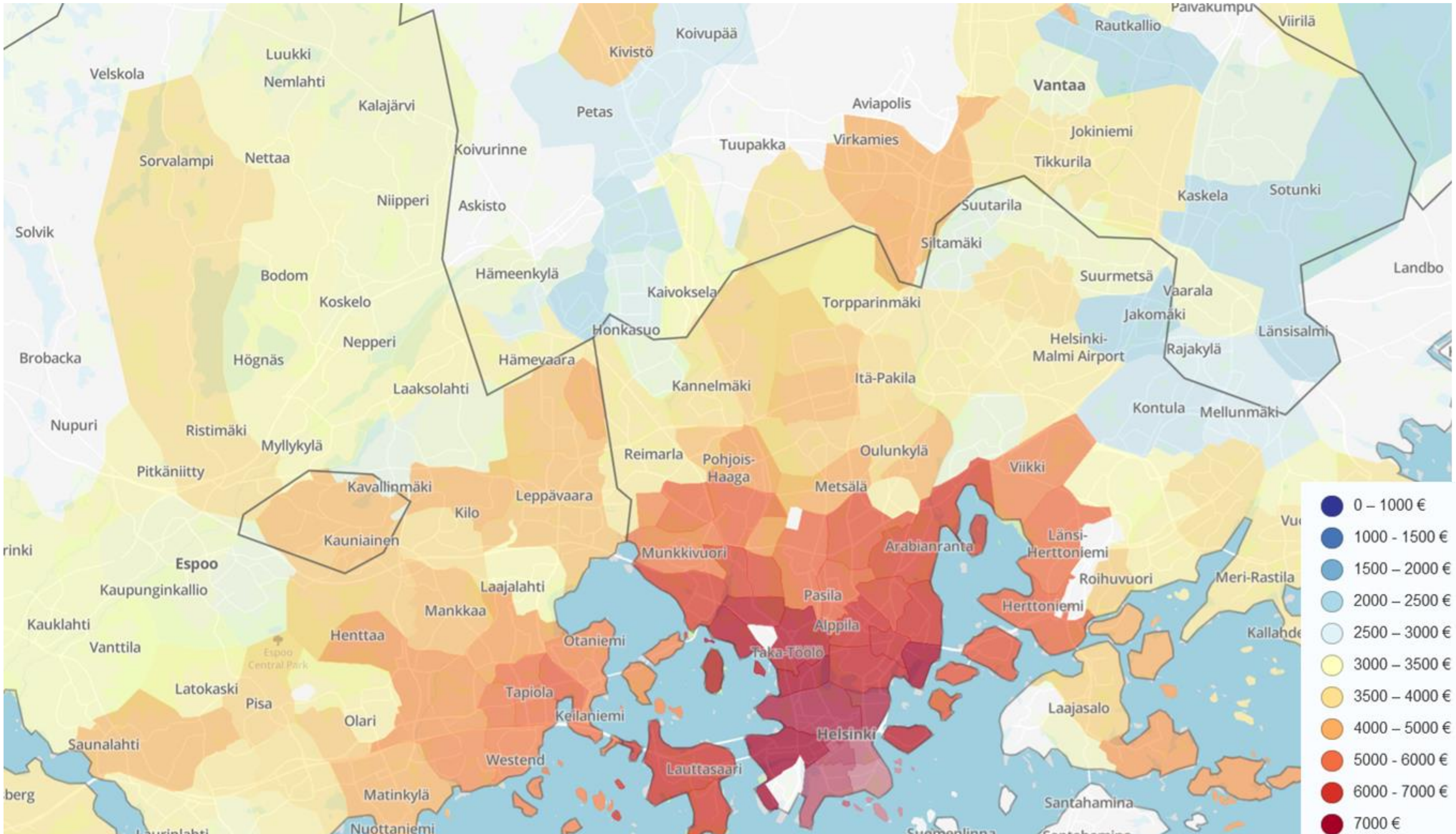
Building height and land prices in Chicago



Source: Ahlfeldt and McMillen (2018), *Review of Economics and Statistics*.

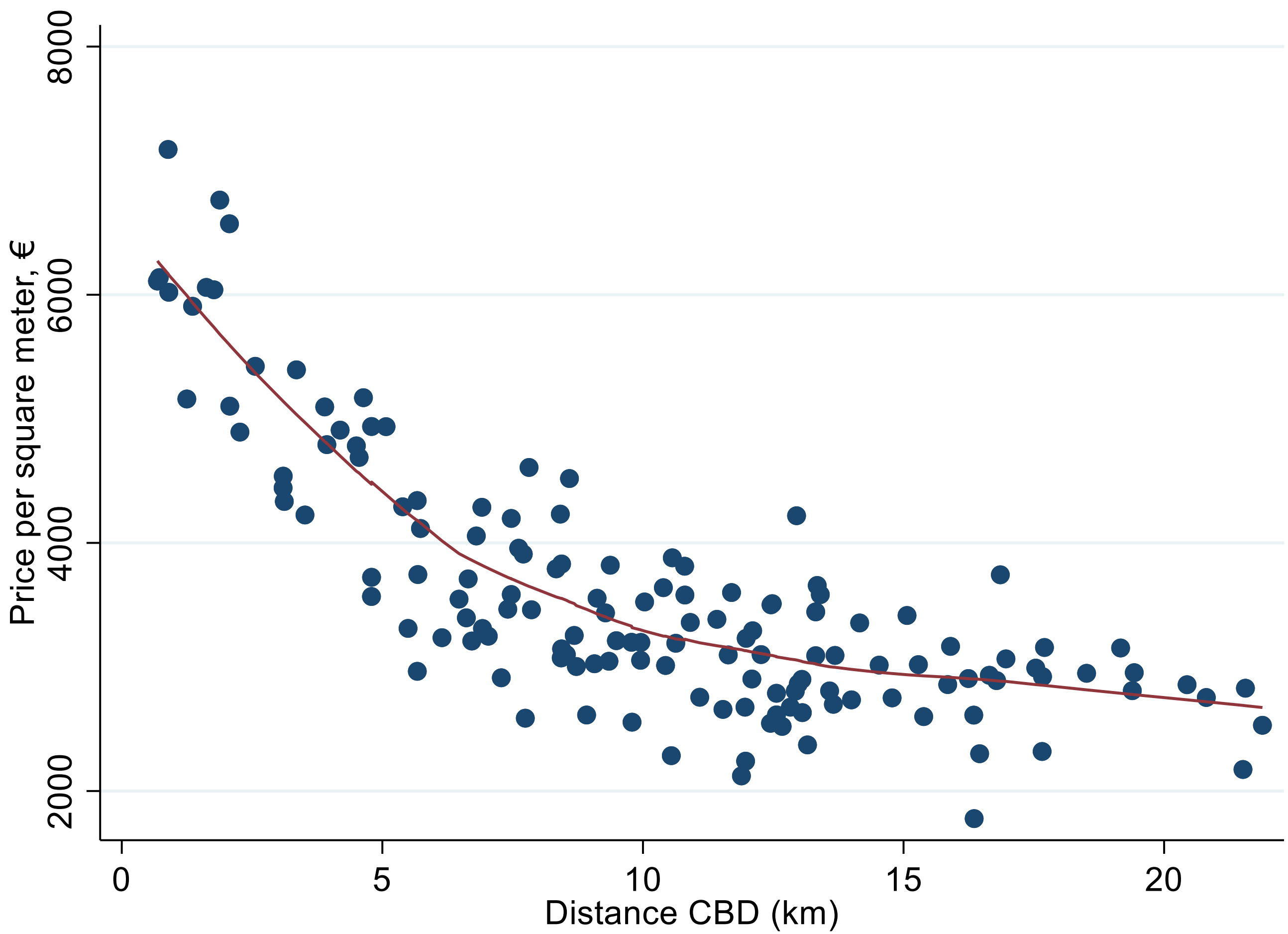
Notes: The building heights in 2014 are from Emporis.com. The 1990 land values are from Olcott's blue books. The y-coordinate is the vertical Cartesian coordinate in the State Plane Coordinate System (Illinois East).

House prices (€/m²) in HMA postcodes

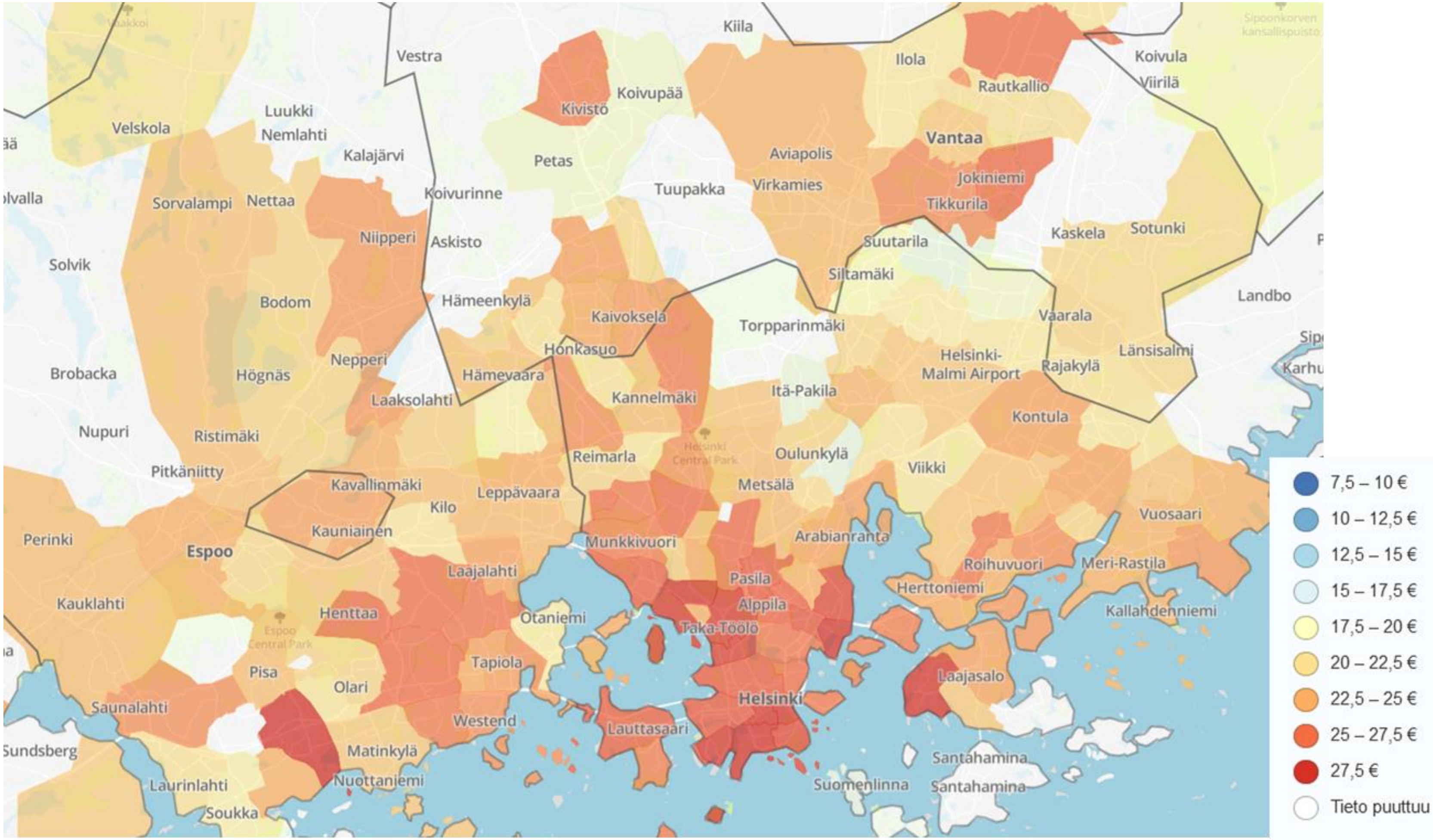


Source: https://asuntojen.hintatiedot.fi/haku/

House prices (€/m^2) in HMA postcodes

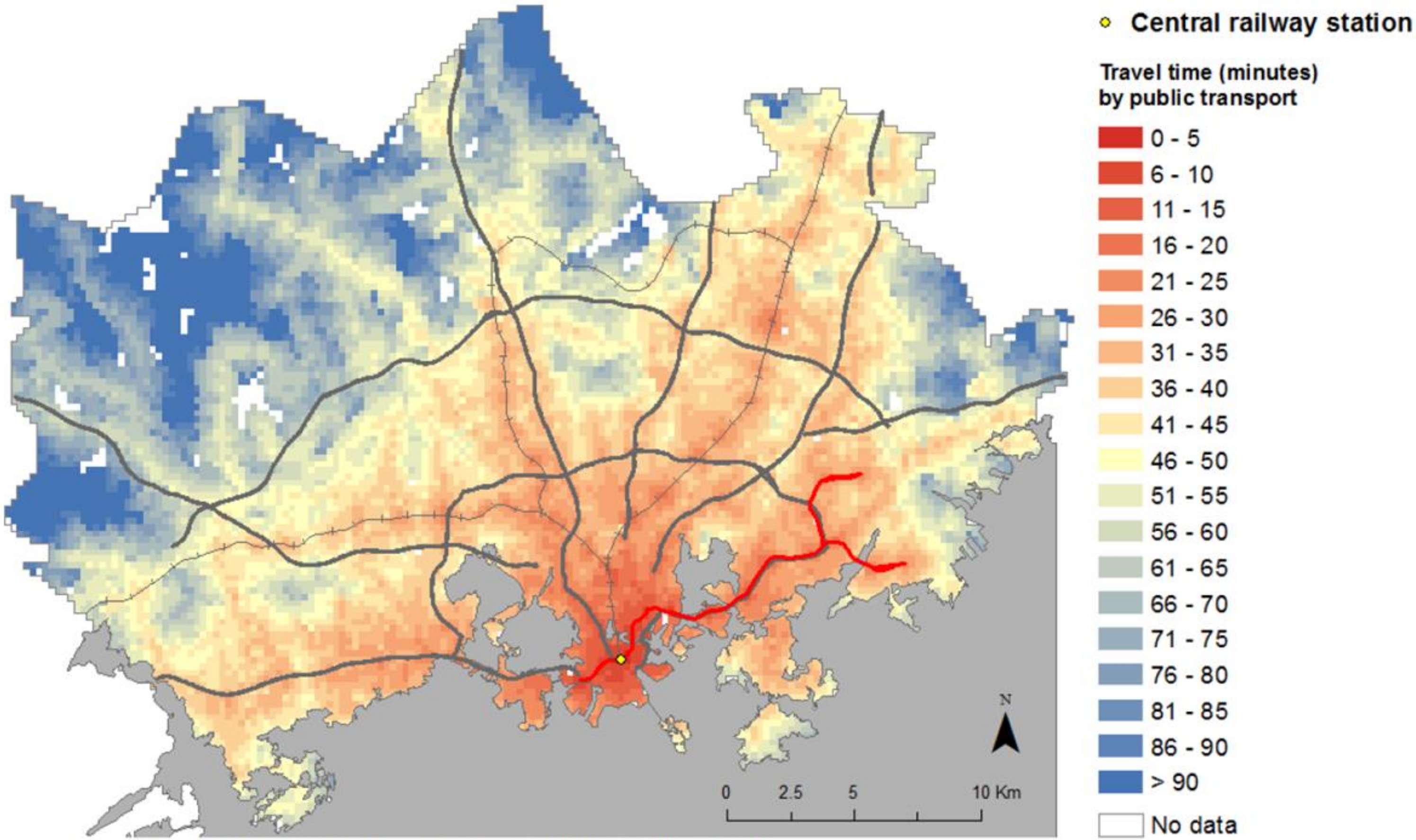


Monthly rents (€/m²) in HMA



Source: https://asuntojen.hintatiedot.fi/haku/

Travel times in HMA



Transport lines: © City of Helsinki, Municipalities in the Helsinki Region and HSY (2015)
Travel times: © MetropAccess-project / Accessibility Research Group, University of Helsinki (2015). License: CC BY 4.0

Patterns

We generally see a pattern of declining density radiating from one center, or sometimes multiple centers

Tall multi-family buildings tend to be located near the city center, while single-family houses are at the fringe

Land and housing prices per square meter/foot tend to be high near the city center and lower farther away

- Spatial equilibrium!

Of course, these patterns are not purely market driven as urban planning has played a major in role

Next, we try to explain these patterns through a simple model

Monocentric city model

Useful concepts – Land

Land rent is the price for using one unit of land, say a hectare, for one unit of time, say a year

Land value is the price of buying one unit of land, again say a hectare

Land is an asset; like any asset its price (= value) is the present value of the benefits (= net rents) from owning it

Determination of land value

Value of a
Land Parcel

Net Rent of
Land in Year 2

$$V_L = \frac{R_{L1}}{(1+i)} + \frac{R_{L2}}{(1+i)^2} + \frac{R_{L3}}{(1+i)^3} + \frac{R_{L4}}{(1+i)^4} + \cdots$$

Discount Rate

$$= \sum_{t=1}^{\infty} \frac{R_{Lt}}{(1+i)^t} \approx \frac{R_L}{i}$$

Assumes R_L is constant over time

Important lesson: cheap land => cheap housing?

The price of land is high in some locations because people are willing to pay a lot for housing or commercial activities at that location

It is not correct to say that the price of housing is high because land is expensive!

Policy question: will we get cheap housing if the municipality sells land to developers at a discount?

Useful concepts – Housing

Housing is measured in units of housing services = *q*

- q = quality-adjusted square meters
- Depends on housing characteristics
- For now, we assume that floor space is the only characteristic

***p* = the price (rent) per unit of *q* per year or month (e.g. 20 €/m^2/month)**

***r* = rent for a housing unit = *pq* (e.g. 20*50 = 1000 €/month)**

- If the unit is a rental apartment, r = contract rent
- If the unit is owner-occupied, r is not observed

Determination of house value

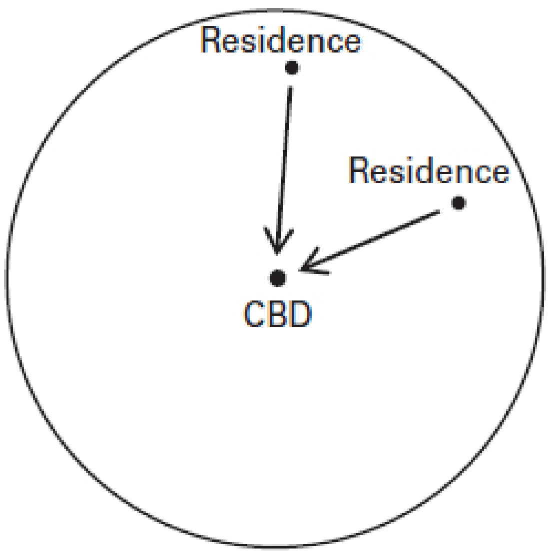
V **=** **value of a housing unit = present value of the rental flow**

So, with a long lifetime, *T,* for housing:

$$V = \sum_{t=1}^{T} \frac{p_t q_t}{(1+i)^t} \approx \frac{pq}{i}$$

Monocentric city model – assumptions

1. **All jobs are in the city center (central business district, CBD)**
   - Jobs do not take up space
2. **The city has a dense network of radial roads**



Monocentric city model – assumptions

1. **All jobs are in the city center (CBD)**
2. **The city has a dense network of radial roads**
3. **The city contains identical households or consumers or workers**
   - Same income/wage (y) and preferences (will be relaxed later)
4. **The residents consume (get utility from) only two goods: housing (q) and a composite good, say bread (c)**
   - The price of the composite good is the same everywhere (equal to 1)
   - Land and the housing that sits on it are allocated competitively to the highest bidder at each location

Commuting costs

The per-kilometer cost of commuting is *t*, so a resident living at distance *x* from the CBD incurs a commuting cost *tx*

- Commuting has only a monetary cost
- Later we will introduce the opportunity cost of time used in commuting
- Also, everyone uses the same commuting mode so that t is the same for everyone

This leaves $y - tx$ for expenditure on housing and the composite good (= disposable income)

- Thus, disposable income decreases as x increases

Housing consumption and budget constraint

A housing unit or a dwelling has variety of characteristics

- Floor space, yard size, construction quality, age, amenities

We simplify things and assume that dwellings differ only in size

- I.e. q represents square meters and p is measured as rental price per square meter

The consumer's budget constraint is $y - tx = pq + c$

- It states that the expenditure on bread and housing is equal to disposable income (income after commuting costs)

Consumer analysis

Consumer analysis

Consumers want to maximize the utility (welfare) they get from consuming housing and bread, while taking into account their budget constraint

That is, the consumer chooses the *c* and *q* to maximize utility *U*(*c*, *q*) subject to the budget constraint at each distance *x*

Location "choice" enters the problem only through commuting costs

- We assumed that dwellings differ only with respect to size, not with respect neighborhood amenities

Price of housing

One of the empirical regularities that we saw earlier was that price per square meter of housing falls as distance to the CBD increases (*p* falls as *x* increases)

Can this simple model predict this regularity?

Yes! And there are several ways to demonstrate this

Locational or spatial equilibrium

Everyone would want live right next to the CBD, but everyone cannot live in the same location

But as consumers are identical, they must be equally well-off regardless of where they live in the city

- If this condition did not hold, then consumers in a low-utility area could gain by moving into a high-utility area (not an equilibrium)

This equilibrium can hold only if price of per square meter of housing falls as distance increases

- Since higher commuting costs mean that disposable income falls as x increases, some offsetting benefit must be present to keep utility from falling

Locational or spatial equilibrium

Lower *p* at more distant locations serves as a compensating differential

- Compensating differentials arise in many economic settings
- For example: dangerous or unpleasant jobs must pay higher wages than more appealing jobs with similar skill level requirements
- Otherwise, no one would do the unpleasant jobs!

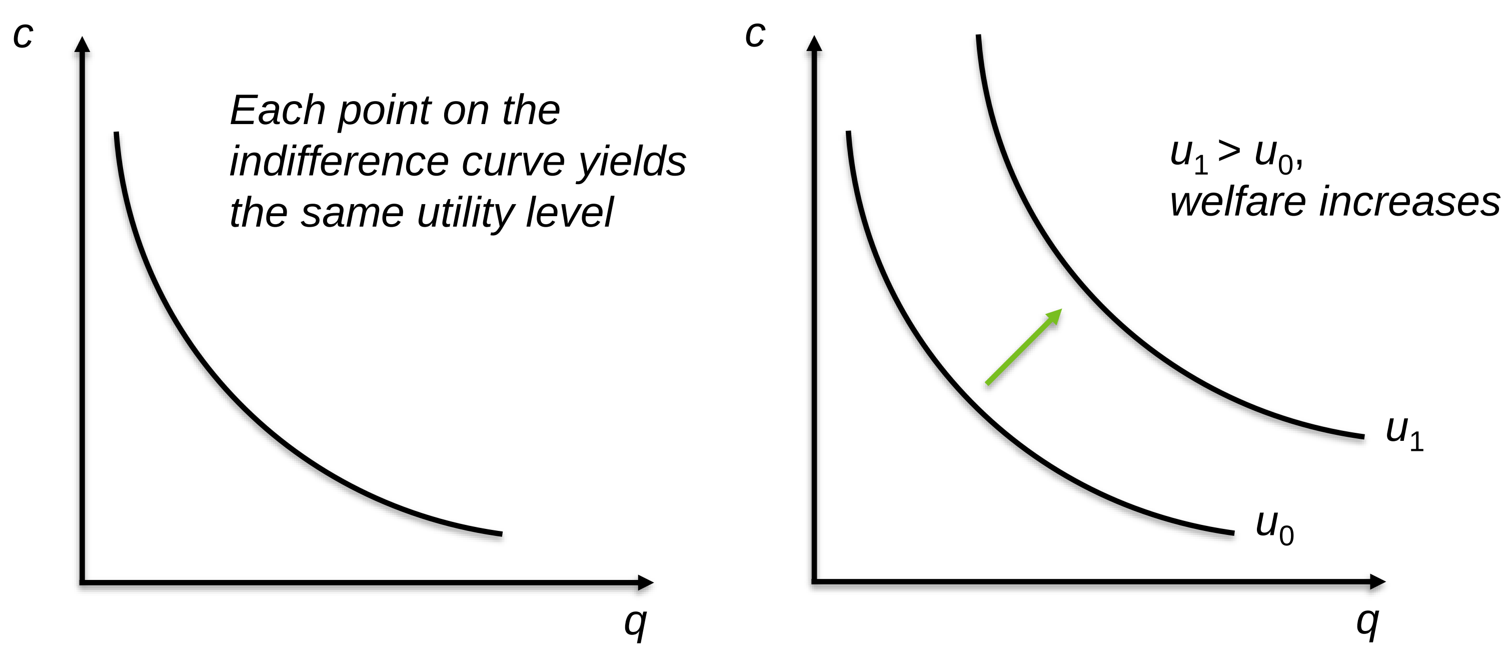
Note that here the price of the composite good is the same everywhere, and thus, cannot play a compensating role

- The prices of groceries and other non-housing goods are the same
- May not be fully realistic, of course

Indifference curve diagram

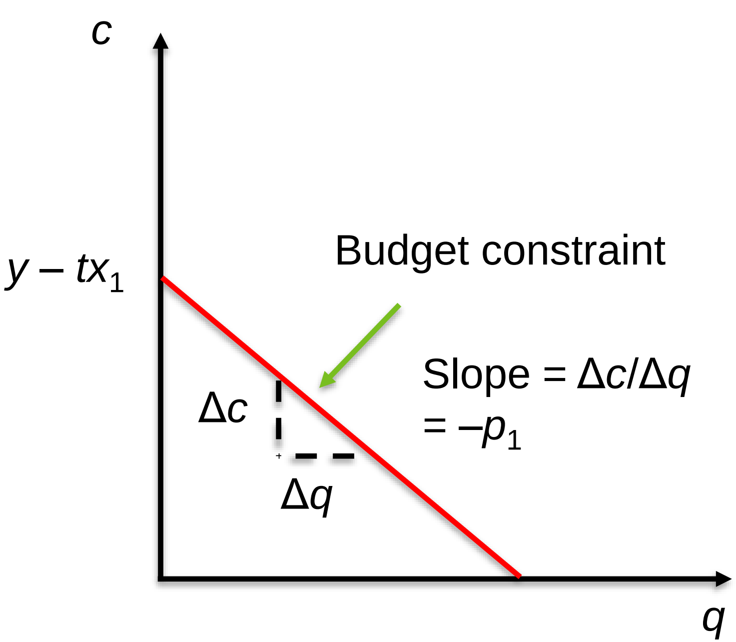
A graphical way of deriving this result is using **indifference curves** and the **budget constraint**

https://mru.org/courses/principles-economics-microeconomics/consumer-choice-indifference-curves-marginal-rate-substitution
https://mru.org/courses/principles-economics-microeconomics/consumer-choice-budget-constraints-opportunity-costs



Budget constraint

$y - tx = pq + c$
$c = y - tx - pq$



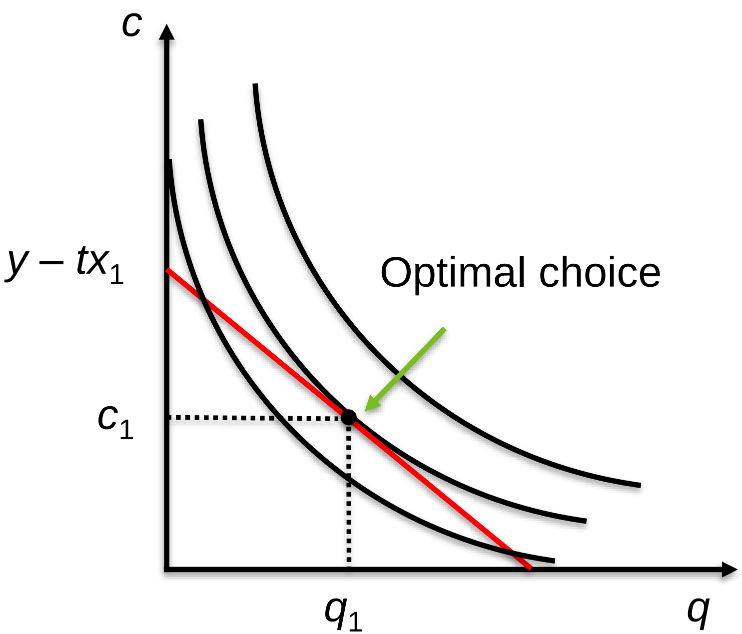
The figure depicts the budget constraint/line for a consumer living at a distance of x_1 away from CBD

If housing consumption is zero, the consumer can consume $y - tx_1$ worth of bread

When the consumer starts to consume housing, it must give up on bread consumption

The slope of the budget line for consumer living at x_1 is $-p_1$

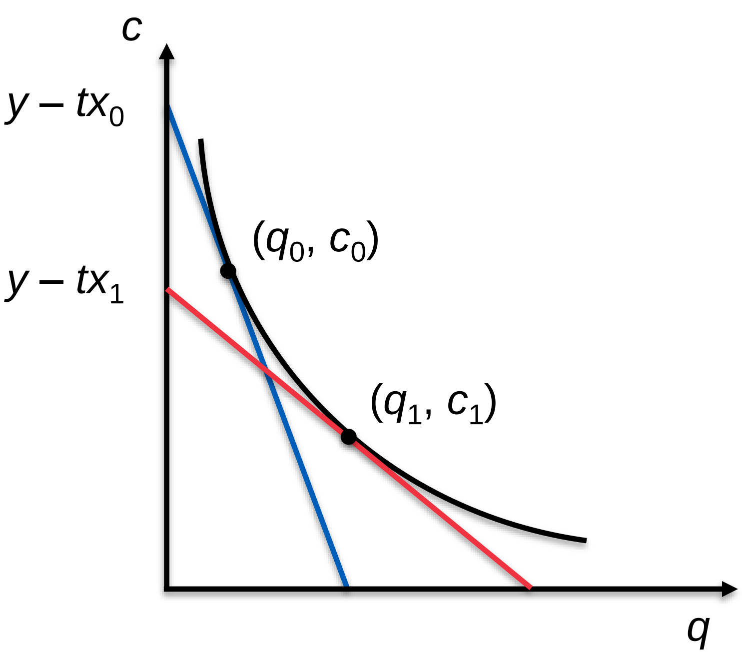
Indifference curve diagram



The consumer chooses the point where the indifference curve is tangent to the budget line (c_1, q_1)

This is the highest possible indifference curve that the consumer can reach within the budget constraint

Central-city and suburban consumer



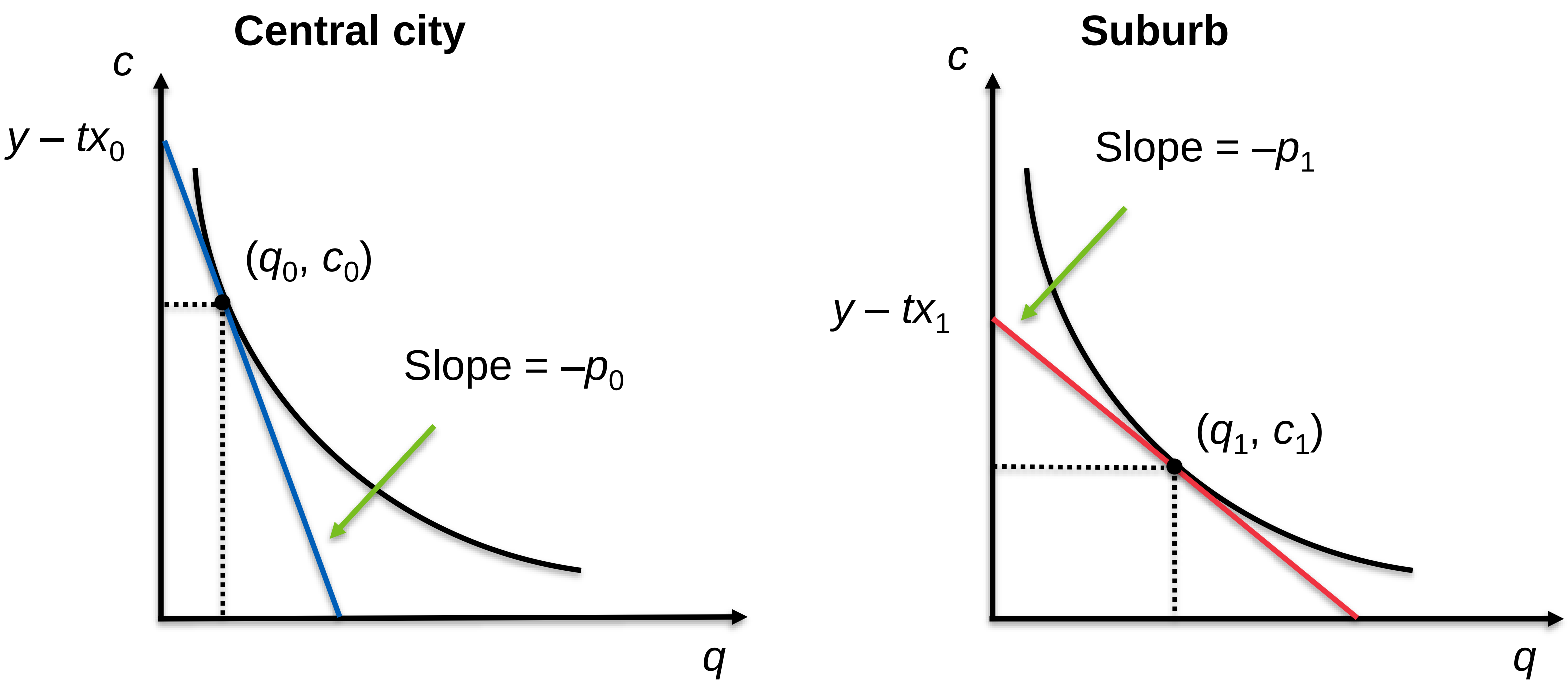
Consider now two consumers, one living central-city (x_0) and the other in a suburban location (x_1), so that $x_1 > x_0$

What magnitude must the price of housing p_1 be at distance x_1 to ensure that the suburban consumer is just as well-off as the central-city consumer?

The price must lead to a budget line that allows the suburban consumer to reach the same indifference curve as the central-city consumer

That is, prices per square meter are higher in central-city, $p_0 > p_1$

Central-city and suburban consumer



Housing consumption in different parts of the city

The diagram reveals another important result of the model

- The suburban resident consumes more housing space ($q_1 > q_0$) and less bread ($c_1 < c_0$) than the central-city resident
- This means that dwelling size *q* increases as distance *x* from the CBD increases

This substitution in favor of housing and away from bread is the consumer's response to the decline in the relative price of housing as *x* increases

- Remember that the price of bread is the same everywhere in the city

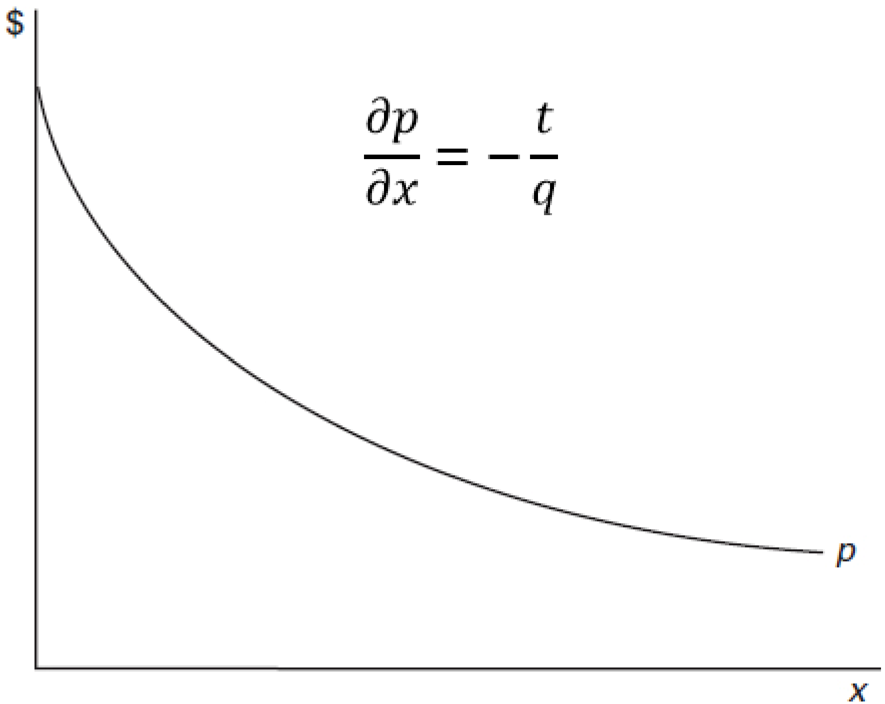
Model predictions

So far, the model's two main predictions are that as distance to the CBD increases

1. Price per square meter of housing falls; $p \downarrow$ as $x \uparrow$
2. Size of the dwellings increases; $q \uparrow$ as $x \uparrow$

Two additional results concern the shape of the curve relating housing price *p* and distance *x* and the total price/rent *pq* and distance *x*

Housing price curve

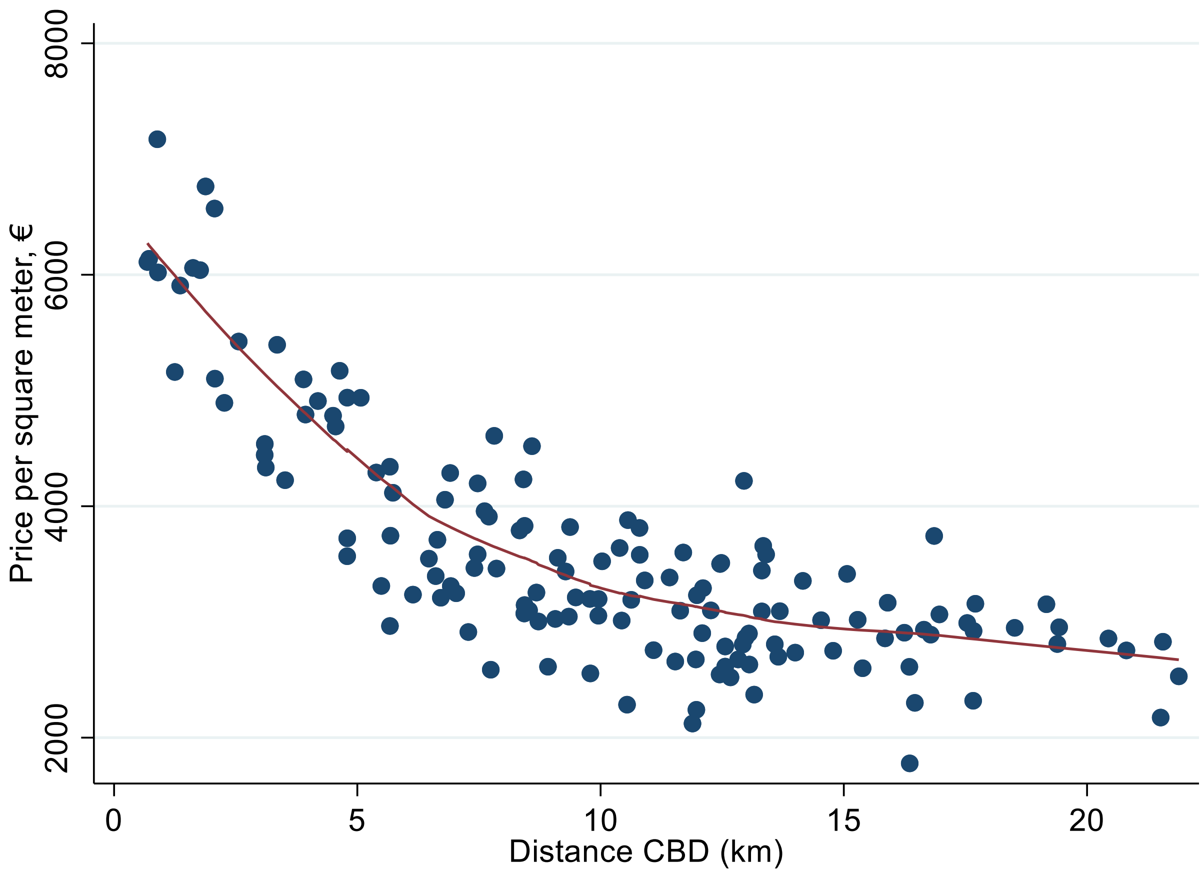


$$\frac{\partial p}{\partial x} = -\frac{t}{q}$$

The price curve is convex if housing consumption increases with *x*

Consumers substitute cheaper housing for bread, so prices do not have to decline as quickly to compensate consumers

Housing price curve in HMA postcodes



Spatial behavior of total rent *pq*

How does the total rent (*pq*) for a small central-city dwelling compare to the total rent of a larger suburban house?

The answer is ambiguous

- Since p falls with x while q increases, the product pq could either increase or decrease
- Which is the case, depends on the consumer's preferences or the shape of the indifference curve

Analysis of housing production

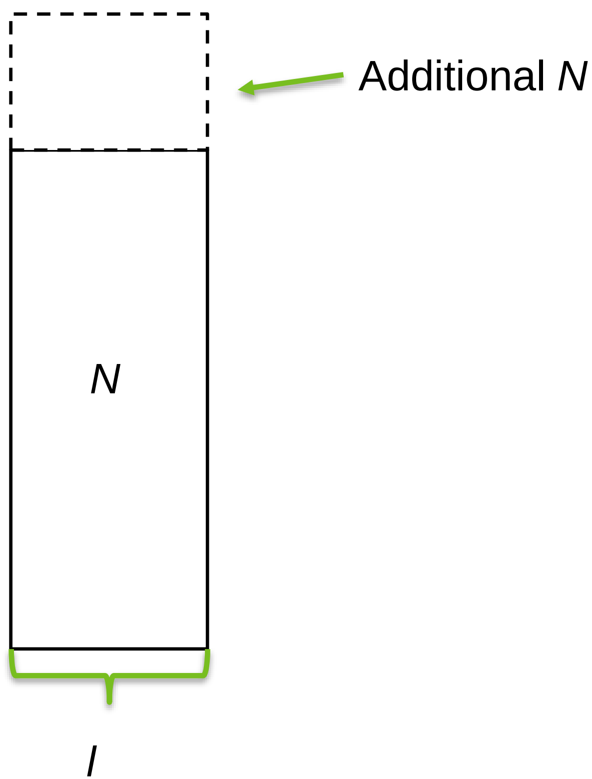
Analysis of housing production

Now we turn to analyzing the production side of the market and focus to the activities of housing developers who build structures and rent the space to consumers

Again, this is a stylized model with several simplifying assumptions

- We assume that housing is produced using only land (l) and building materials (N) (we refer to N as capital also)
- The production function for housing is $Q = H(N, l)$, where Q is the amount of floor space in the building (q is dwelling size!)

Diminishing marginal return to capital



With land input (size of the lot) held fixed, extra doses of building material lead to smaller and smaller increases in floor space

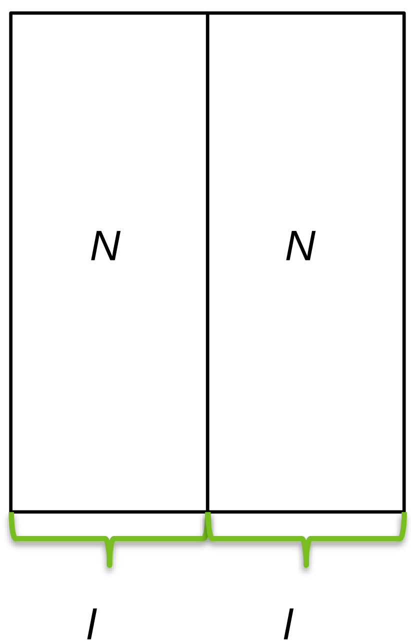
This makes sense as increasing N with fixed l makes the building taller

Stronger foundation, thicker beams, elevators…

We are assuming that the building completely covers the land area, so there are no yards

Again, this assumption can be relaxed with the of price of complicating the model

Constant returns to scale

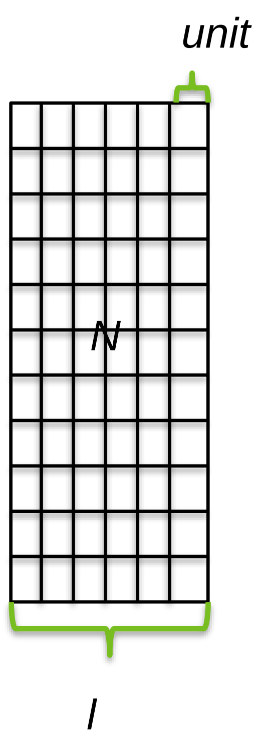


Economies of scale are present when doubling both the capital and land inputs leads to more than a doubling of the floor space

The figure suggests that doubling both inputs leads to exactly doubling of floor space

Thus, we assume that housing production exhibits "constant returns to scale", at least approximately

Profit maximization



The housing developer chooses the capital and land inputs to maximize profits, which leads to a building of particular height

The developer also implicitly chooses the amount and size of the dwellings in the building

The latter decision simply responds to consumer choices, i.e. the demand side of the market

Profit maximization

The developer's revenue from a building is pQ or $pH(N, l)$, where p is the price per square meter of housing as before

Input costs come from capital and land

We assume that the developer rents the land and capital inputs

- Land rent per square meter is denoted with r and rent for capital with i
- The price of capital is assumed to be fixed, i.e. there are no differences in physical building costs within the city

Production costs equal $iN + rl$, i.e. capital costs + land costs

Equal profits across space

In consumer analysis, the utility or welfare of all consumers was the same everywhere in the city

We have a similar spatial equilibrium condition on the producer side: profits are equal everywhere in the city

- If not, developers would not be willing to build housing everywhere
- Because i doesn't vary with location, it is the spatial variation in land rent r that equates profits and makes developers willing to build housing throughout the city

Land rent as a compensating differential

Central-city locations offer higher revenue per square meter than suburban locations

This means that land rents must be lower in the suburbs than at central locations

With r falling as distance x increases, the disadvantage of lower revenue is offset

Land rent as a compensating differential

We can also think about this as a demand-based phenomenon

- Developers compete for prime locations where housing prices are high
- This higher demand for land and competition among developers bids up land rents near the CBD
- There is less demand for remote lots and land rents will be lower farther away from the CBD
- Competition for prime locations drives land rents so high that uniform profits across space prevail (normal economic profit)

Building height and distance

With the price of capital fixed and land rent rising moving toward the CBD, the land input becomes more expensive relative to capital as distance to CBD declines

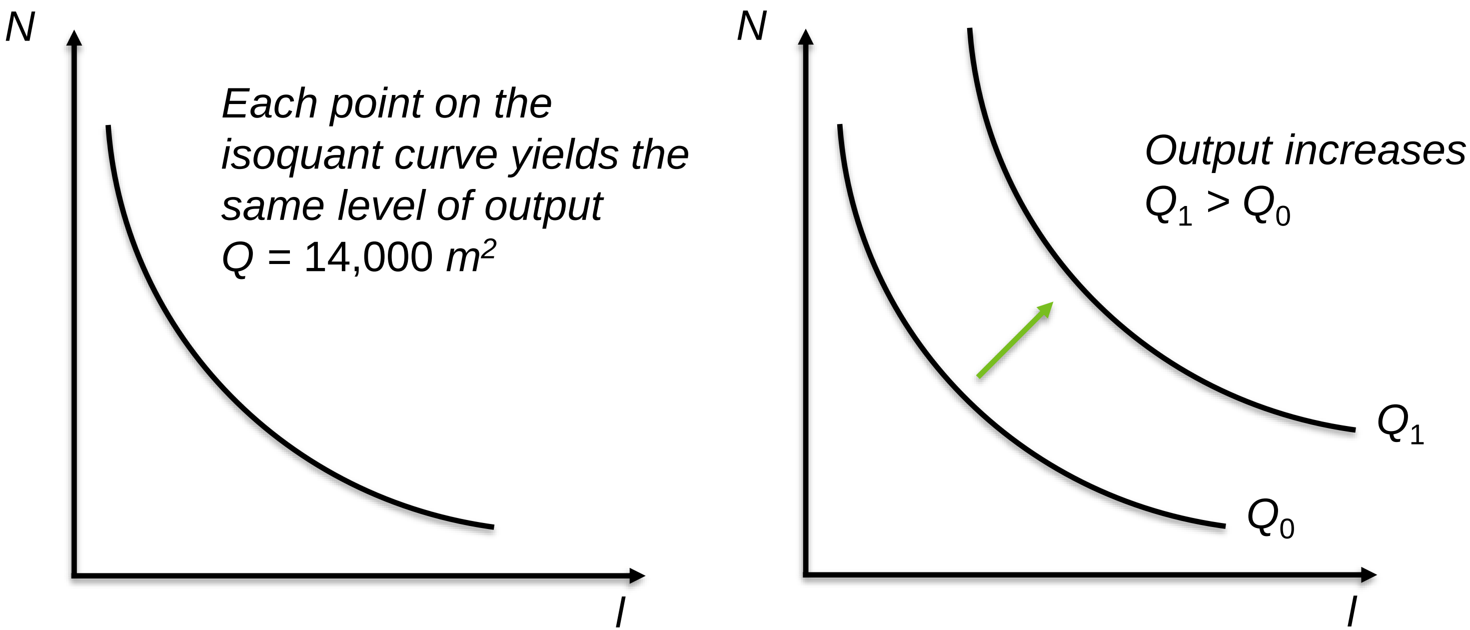
- This incentivizes developers to economize on land input and use more capital which leads to taller buildings
- Conversely, as land gets cheaper moving away from CBD, developers use more of it and build shorter buildings

Overall, building height decreases as distance to the CBD increases

Isoquant curve diagram

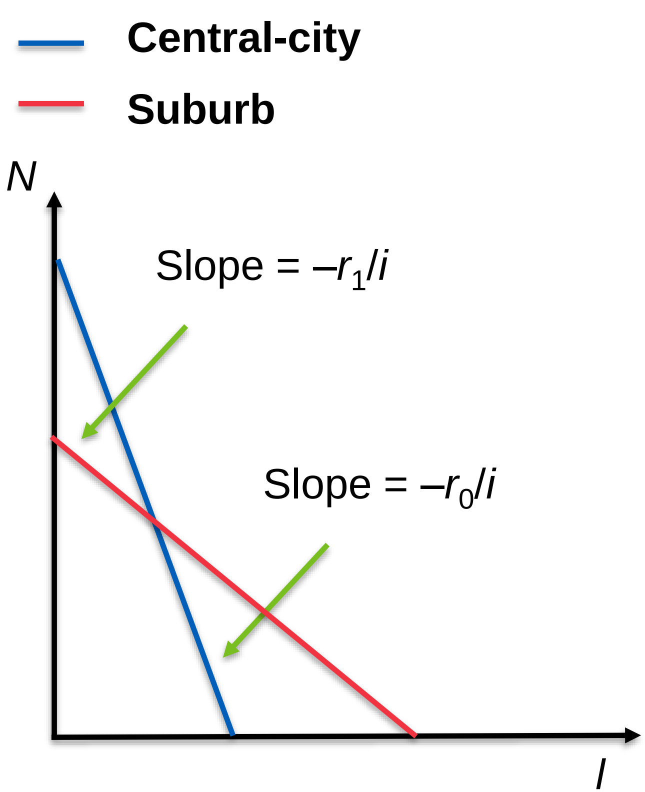
A graphical way of deriving this result is to use a diagram illustrating cost minimization on the part of the developers

Isoquant curves that show the combinations of inputs that yield the same output



Iso-cost lines

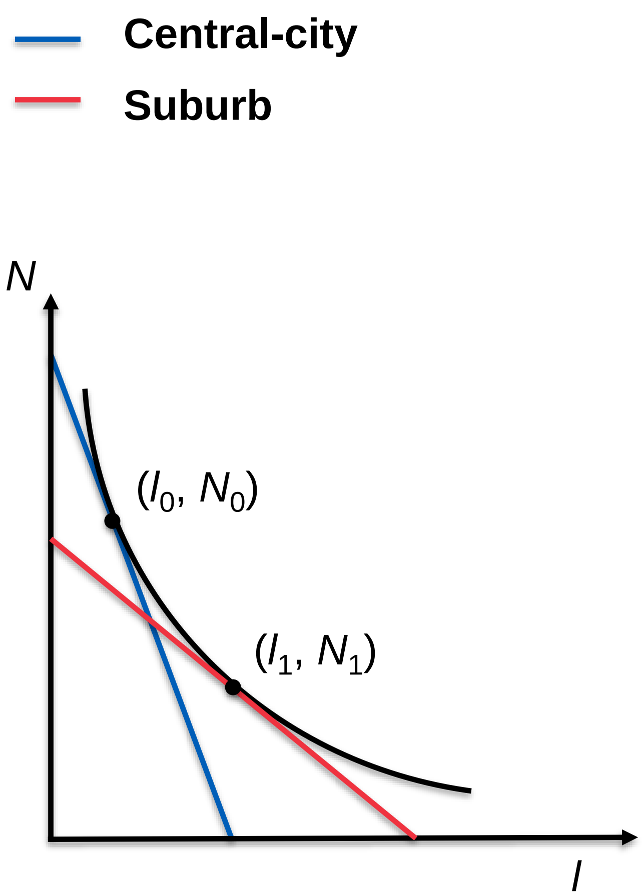
Fix $iN + rl$ to some number



The figure depicts two iso-cost lines

- Iso-cost line = combination of inputs that cost the same total amount
- Central-city developer at a distance of x_0 away from CBD with land rent r_0
- Suburban developer with x_1 and r_1
- The slopes of the iso-cost lines are $-r_0/i$ and $-r_1/i$, respectively
- The slope for the central-city developer is steeper, because r_0 is high
- I.e. the central-city developer must give up more N in order to acquire additional units of land (iN buys you less l)

Central-city and suburban developer



To produce Q square meters of floor space as cheaply as possible, each developer chooses the input bundle on lowest possible iso-cost line

Developers facing different land rents (but same capital rents) use different amounts of inputs

Since the central-city developer is using more capital and less land, the central-city building is taller than the suburban building

Predictions from producer analysis

Two main predictions are that as distance to the CBD increases

1. Price per square acre of land falls; $r \downarrow$ as $x \uparrow$
2. Building height decreases; building height $\downarrow$ as $x \uparrow$

Population density

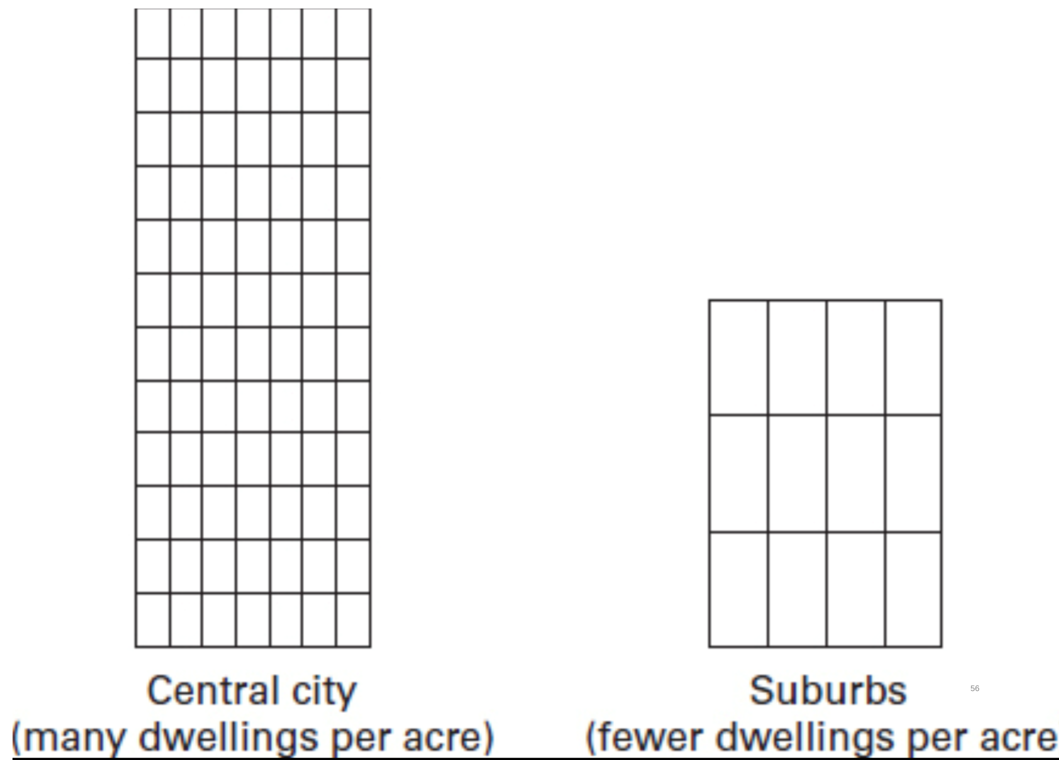
Combining the consumer and producer analysis yields a further result regarding population density in different parts of the city

- Population density (D) is measured as the number of people per km^2
- Central-city location has tall buildings divided into small dwellings, while the suburb has short buildings divided into larger dwellings
- This implies that population density is higher in the central-city

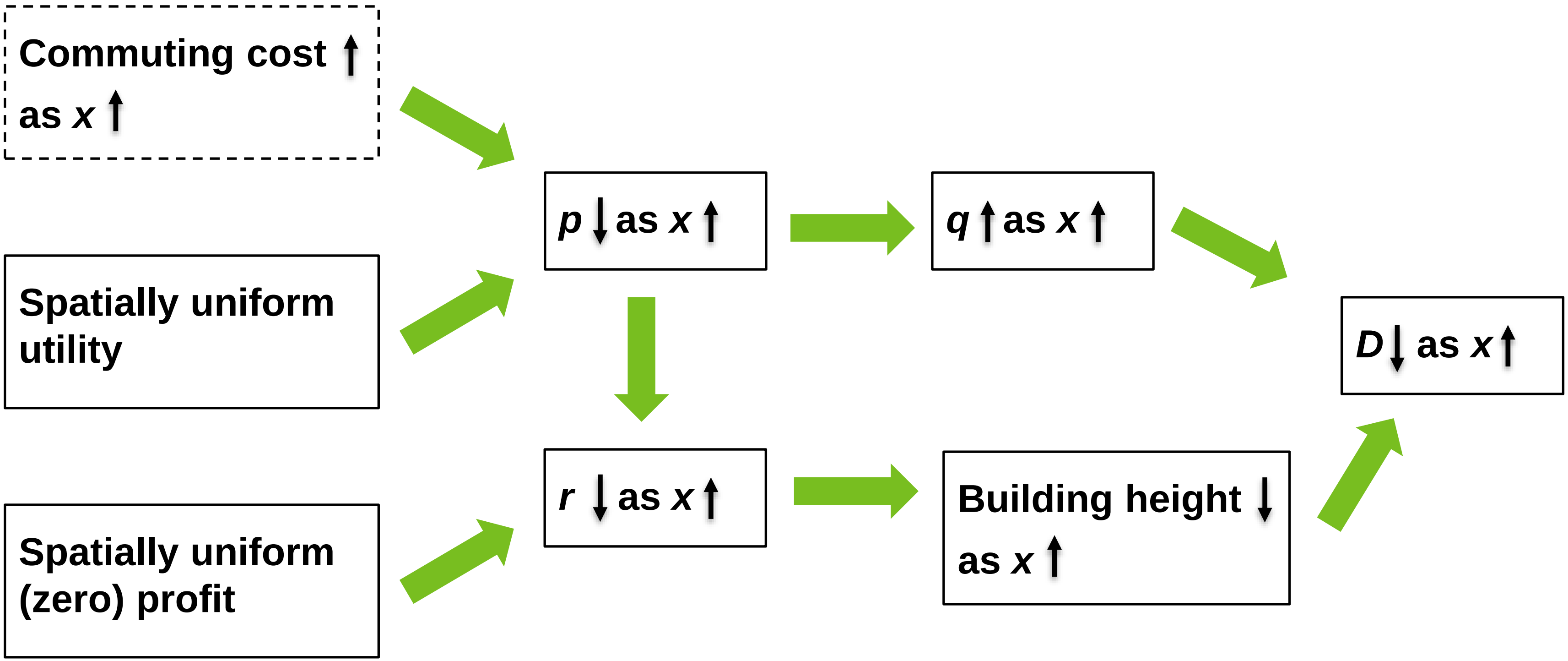
Thus, *D* falls when moving away from CBD; $D\downarrow$ as $x\uparrow$

Population density

Buildings and dwelling sizes in central-city and the suburb



Summary of the model logic



Empirical example – New metro line

OSKARI HARJUNEN

METRO INVESTMENT AND THE HOUSING MARKET ANTICIPATION EFFECT

Do people value accessibility and by how much?

One of the central predictions of the model was that house prices per square meter are higher in locations closer to the CBD with low commuting costs

But the model is very stylized. In reality, locations differ in several other ways besides accessibility

How can we know whether and by how much people really value accessibility?

Solution: look at how prices change when accessibility changes and compare this to price changes in places where accessibility does not change (DID)

West Metro extension

Harjunen (2018, chapter of his PhD thesis) analyzes the price effects of the West Metro extension in the HMA

- https://www.hel.fi/hel2/tietokeskus/julkaisut/pdf/18_01_25_tyopapereita_02_Harjunen.pdf

The West Metro introduced eight new metro stations – two in Helsinki and six in Espoo

The study focuses on the time period when the construction of the new line started, but before it became operational

The idea is to see whether the extension is anticipated in the housing market already before the new line was operational

Research design

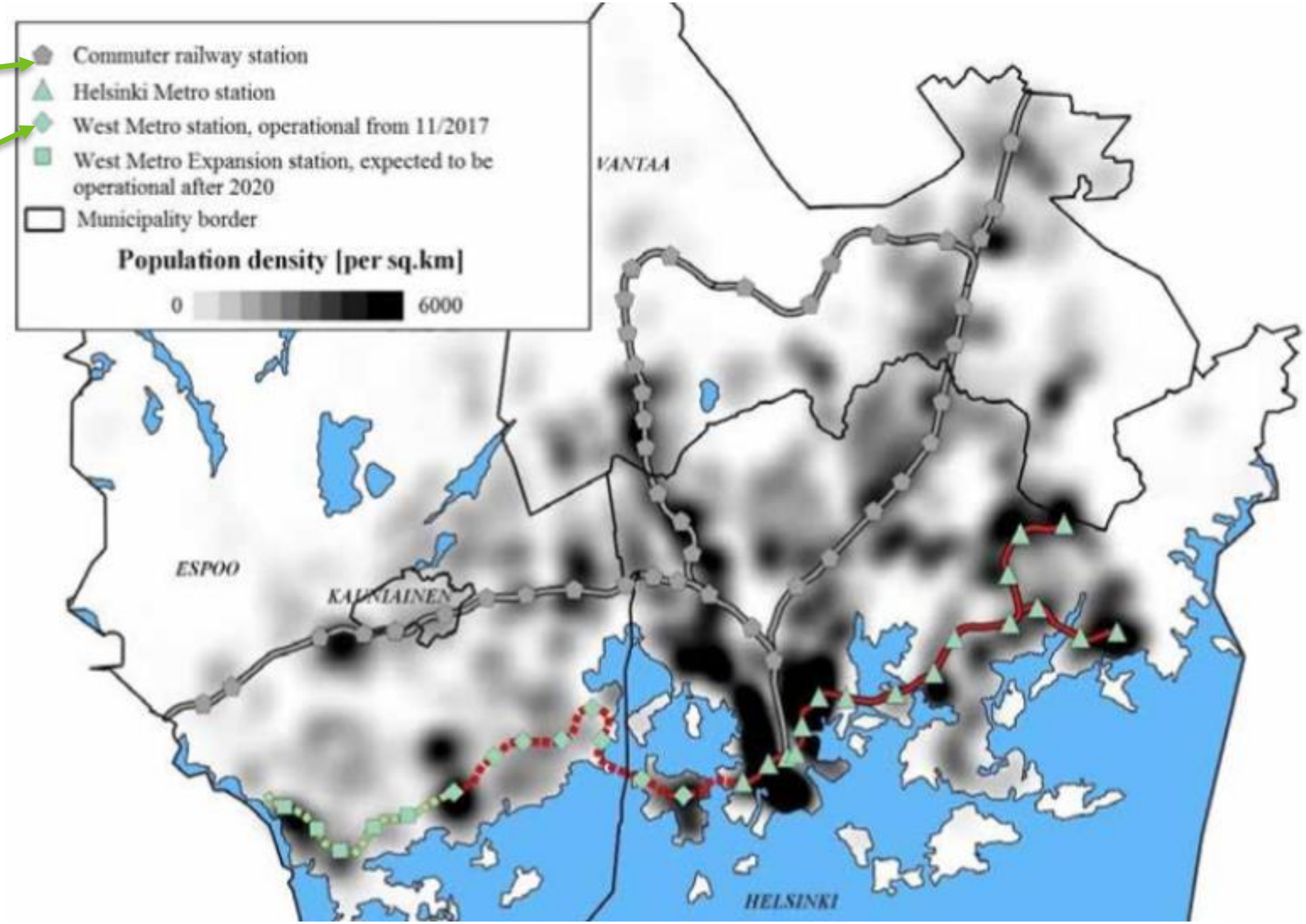
Fig. A2. A route map of the metro in Helsinki in 2016 (© HSL 2016)

Fig. A3. A route map of the metro in Helsinki and Espoo after west metro is operational in 2017 (© HSL)

Research design

Control

Treatment



Timing

The construction of the West Metro was finally approved in the city councils of Espoo and Helsinki in September 2008

The underground master plan of the West Metro was approved in January 2009 and the official ceremony initiating the construction works took place 11th of November

- But in large scale the constructions began in 2010 after delays caused by the appeal process

In the study the "treatment period" begins at the start of 2010

At this time, it was clear where the new metro stations will be located

Data

Data comprises of transactions in Helsinki and Espoo from 2003 to 2016

The data are collected by a consortium of Finnish real estate brokers and the dataset is refined and maintained by the Central Federation of Finnish Real Estate Agencies (KVKL)

- As not all real estate agencies participate, the dataset represents a sample (albeit rather large) of the total volume of transactions

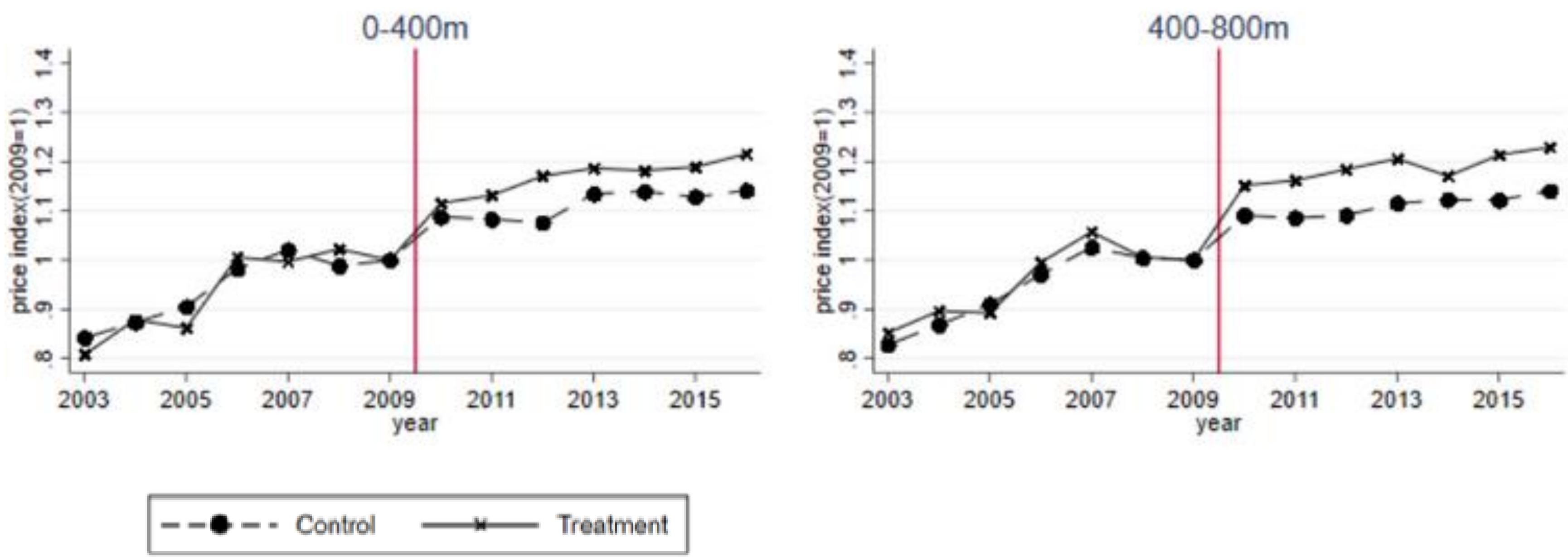
The data include the transaction price and sale date for each dwelling as well as a rich set of dwelling characteristics including its exact location

- The sample is restricted to multi-story and row house sales

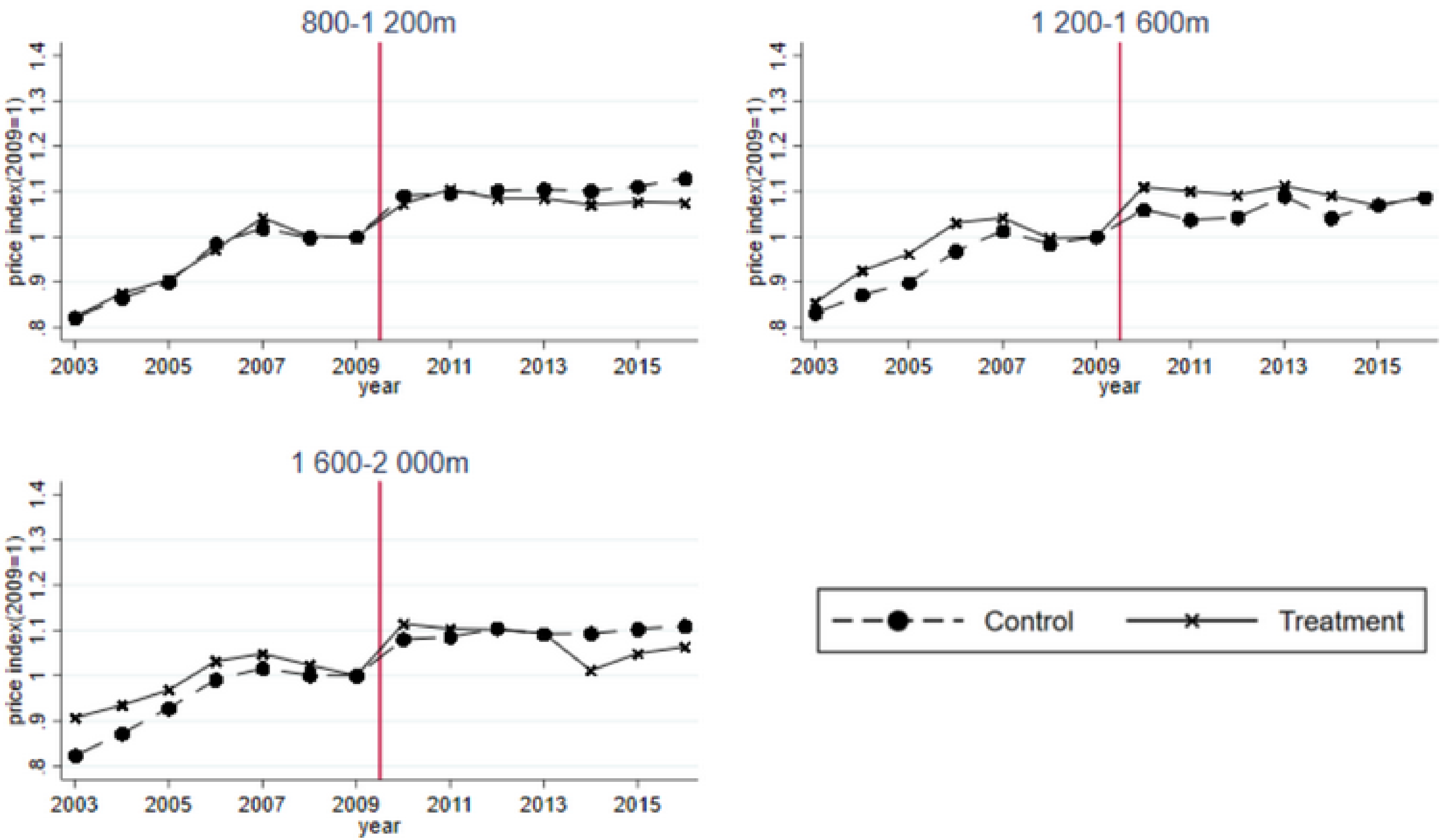
Data

Sample	Whole data (Helsinki and Espoo)	0 to 800m		800 to 1 600m	
Status		Treated	Control	Treated	Control
N	43 025	6 868	15 640	4 429	11 267
Sale price	223 668	252 024	196 154	311 661	199 122
	[110 007]	[119 458]	[78 980]	[156 343]	[82 107]
Square price	3 506	4 181	3 325	3 877	3 242
	[918]	[951]	[805]	[919]	[805]
Area	66	62	61	82	64
	[29]	[27]	[25]	[38]	[27]
Age	37	43	32	32	39
	[17]	[17]	[17]	[13]	[18]
Maint. Charge (€/m2)	3,5	3,8	3,5	3,5	3,5
	[1.2]	[1.1]	[1.2]	[1.2]	[1.3]
Floor number	2,4	2,7	2,5	2,3	2,3
	[1.6]	[1.7]	[1.5]	[1.5]	[1.4]
Floors in building	3,8	4,4	3,8	3,6	3,4
	[3.0]	[2.2]	[2.1]	[2.3]	[1.9]
Dist. to nearest station (m)	869	482	484	1 168	1 134
	[489]	[190]	[185]	[239]	[239]
Dist to CBD (km)	12	9	13	11,2	12,5
	[4.6]	[3.6]	[4.8]	[3.2]	[4.6]

Results – graphical

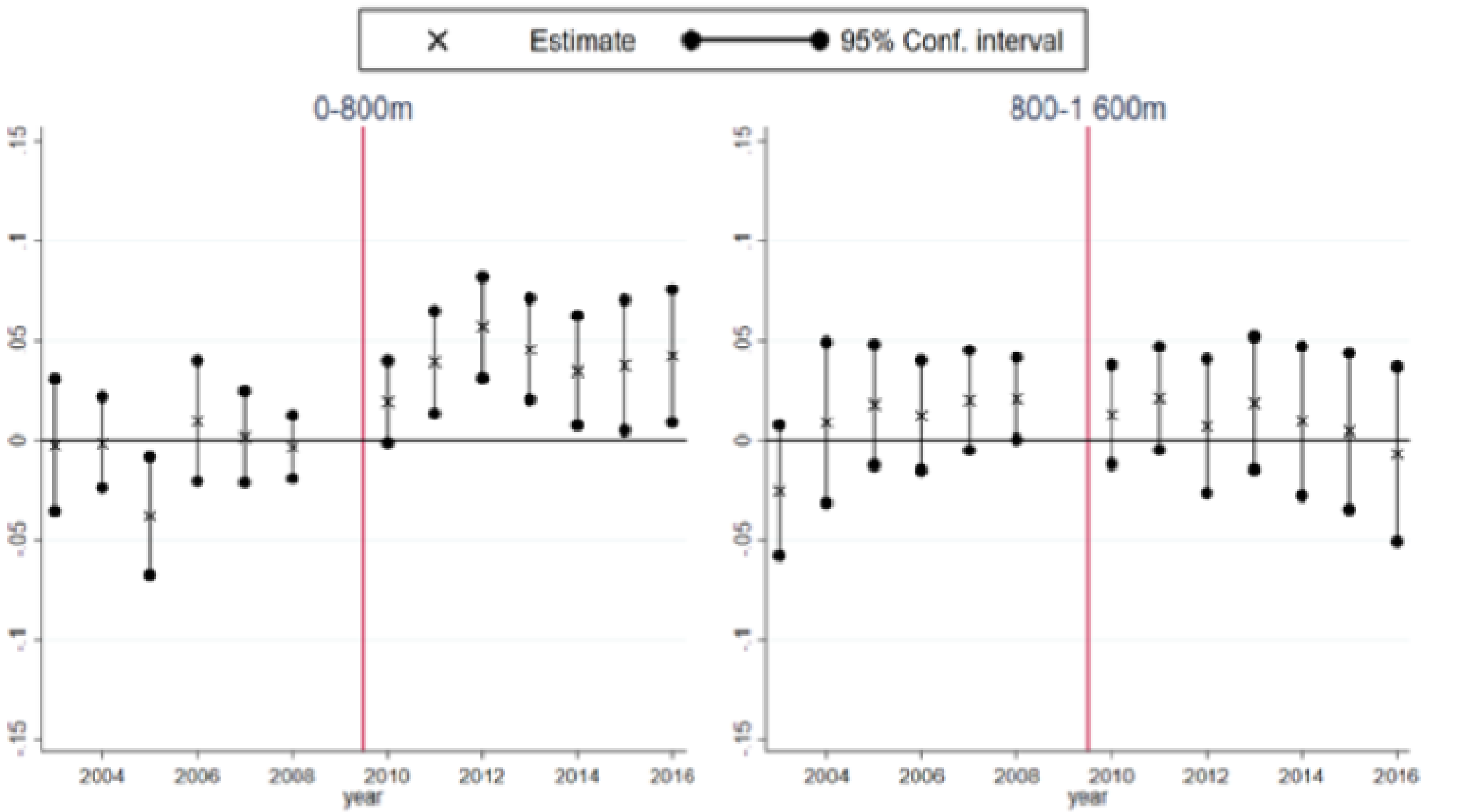


Results – graphical



Results – regression model

Fig. 3. Coefficients of yearly estimates, year 2009 omitted



Conclusion from Harjunen (2018)

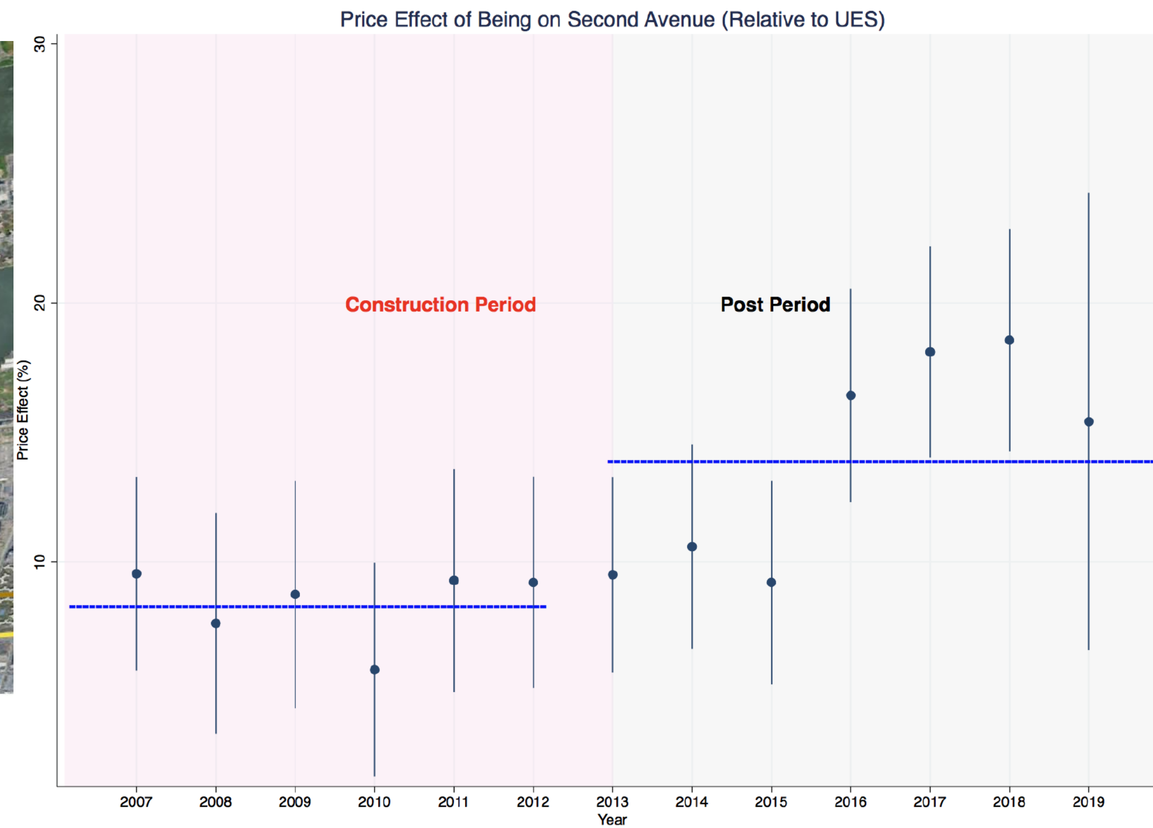
Housing markets start adjusting to the information about the infrastructure investment swiftly after the construction begins, years before the line becomes operational

Apartments within 800 meters from the new metro stations, where the accessibility will be increased the most, experience a positive price increase that converges to around 4%

Questions:

- What are the likely further effects of this price increase in the old housing stock?
- What are the implications for the interpretation of the results

Gupta, Van Nieuwerburgh and Kontokosta (2022): New subway line in NYC



Link to paper: https://www.sciencedirect.com/science/article/pii/S0094119021001042