Lecture notes on electricity markets

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1 Introduction

These notes explain how the market equilibrium in the day-ahead electricity markets is solved and what are some of the key drivers for equilibrium outcomes. After a brief description of the market institutions, we start with the perfect market ideal: how to interpret the way in which market equilibrium is solved in terms of surplus maximization in perfectly competitive markets. Examples from California, the Nordic area, and Spain are used to illustrate the similarities and differences in the institutional set-up and the way in which the available generation capacities affect the market outcomes. This note builds on the general economics overview of electricity markets given in Green (2005), but we start from a different angle.

2 Market institutions

The wholesale electricity trade is set up in a fundamentally similar manner in most regions where electricity markets have been opened for competition. A market operator arranges a day-ahead uniform price auction for each hour of the following day to match demand and supply bids of generators and retailers[1]. The technical requirements for keeping the lights on require also a more real-time matching of supply with demand, but the wholesale day-ahead trade is still economically an order of magnitude more valuable.

Consumers' electricity demand is served by the retail companies who set the tariffs to their customers. Traditionally these tariffs have been based on average consumption over time spans dependent on how often the meter has been read, but with smart meters the measurements are taken hourly or even more frequently; enabling dynamic pricing and incentives for more flexible consumption patterns.

In the course examples we will focus on three regions, California, the Nordic market, and Spain, so a brief introduction to the markets is given here. Although we aim for more general understanding of connection between the economics and the environment, these electricity markets provide a rich environment and an important industrial sector on their own to warrant closer scrutiny.

In each of the regions, the day-ahead markets for electricity are well-established; they have been in place already for around two decades. Although the markets are of similar size, the mix of production technologies, and resulting wholesale market environments differ. The Nordic market has the largest consumption because of the large energy intensive industry sector and the wide-spread use of electric heating. The high share of low-cost renewables and nuclear lead to low wholesale prices. Of note is the role of hydro power in the Nordic region; with over half of the total generation, hydro power gives a significant source of flexibility in the supply side. In California, lower natural gas prices keep the wholesale market prices lower compared to Spain, where the wholesale prices and euro value of the market is higher. In all regions, the retail prices facing consumers are many times higher than the wholesale prices, the large gap is explained by distribution and transmission costs, taxes (including VAT) and levies, and margins

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[1] The origins of electricity markets are set by the definitive Schweppe et al. (1988).

for the retailers.

Table 1 provides a snapshot of the common characteristics of markets in California, Nordics, and Spain. The markets are in the same ballpark in size, with wholesale market value between 8 to 14 billion U.S. dollars/euros and electricity generation between 206 and 397 TWh per year. All markets have integrated an over time increasing share of solar and wind production, with the current share of intermittent renewable electricity generation between 10 % to 20 % of the total annual generation. On the demand side, the number of households ranges from 12 to 18 million, with over 80 % share of "smart meters" that allow for hourly measurement of consumption and prices. Metering as such does not lead to elastic consumer demand, as households' consumption and storage technologies do not change overnight. So far, there is limited evidence of significant changes in elasticity of demand in these markets.

The markets do differ in several technicalities relating e.g. to the real-time maintenance of system operation, the control of market power in the bidding process, the way in which the location of the bids is accounted for, and the acceptance of more complex bid structures, some of which is described in the Appendix. We can safely abstract away from most of such technicalities, they are not essential for the purposes of this course.

3 Competitive equilibrium

3.1 Uniform price auction

In the following, we will go through an equivalent of the market clearing routines operated by the power exchanges, for example the market operators in California, the Nordic market, and Spain, to compute market equilibria. The purpose is to make exact the motivation of setting up the market institutions in the way they are. The exchanges themselves make the claim that the "The price formation process is therefore economically effective for society."[2] Is there any warrant for such claims?

First, let's assume that we have a group of individuals interested in buying electricity. Label these individuals with $i = 1, 2, \ldots, n_i$. Each of the buyers can submit a one purchase bid. A bid consists of the maximum price buyer i is willing to pay, p_i, and the maximum quantity the buyer is willing to purchase, q_i. Similarly, on the supply side, take a group of sellers, $j = 1, 2, \ldots, n_j$. A sales bid consists of the minimum price seller j wants get to sell the good, p_j, and the maximum quantity the seller is willing to sell, q_j.

Given such bids, the following suffices for (P^*, Q^*) to be a competitive equilibrium:

- All buyers whose valuation is higher than the market price P^* purchase the good. All buyers whose valuation is lower than the market price do not.
- All sellers whose cost is lower than the market price P^* sell the good. All sellers whose cost is higher do not.
- The amount of the goods sold equals the amount of the goods bought (Q^*).

If we further assume that the bids that the market participants submit are based on their true valuations for the good on the demand side and the true marginal costs on the supply side, then the market equilibrium will be perfectly competitive.

In a day-ahead electricity market setup, given a set of demand bids, $(p_i, q_i)_{i\in\mathcal{D}_\sqcup}$, and a set of supply bids, $(p_j, q_j)_{j\in\mathcal{S}_\sqcup}$, for any given hour t, we can solve the following optimization problem hour-by-hour:

$$\max_{d_i, s_j} \sum_{i\in\mathcal{D}_t} p_i d_i - \sum_{j\in\mathcal{S}_t} p_j s_j \qquad \text{(OP)}$$

$$\begin{aligned} \text{s.t.} \quad d_t &= \sum_i d_i, \quad 0 \le d_i \le q_i,\ i \in \mathcal{D}_t \\ s_t &= \sum_j s_j, \quad 0 \le s_j \le q_j,\ j \in \mathcal{S}_t, \\ d_t - s_t &= 0. \end{aligned}$$

where d_i and s_j are the quantities of accepted bids, and d_t and s_t are sums over the

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[2] A quote from Nord Pool website.

Table 1: Summary statistics

		California	Nordic	Spain
Electricity generation	TWh	216.7	397.4	264.4
Solar	TWh	33.7	0.9	13.1
Wind	TWh	14.0	33.4	47.7
Hydro	TWh	43.1	222.5	42.6
Wholesale market value	billion \$/€	8.1	11.7	13.8
Wholesale market price	c/kWh	3.76	2.94	5.22
Residential electricity tariff	c/kWh	19.65	20.85	23.83
Number of households	million	14.7	11.8	18.3
Share of smart meters	%	85	80	91

Notes: Summary statistics for the markets in 2017. Sources: EuroStat, EIA.

cleared bids. The market price P^* is given as the shadow price of the balance constraint of supply meeting demand. One way to think of this is that both increasing the demand slightly or decreasing the supply slightly give the same benefit in the maximization problem, i.e. the balance constraint is binding where the marginal value of increasing demand matches the marginal value of decreasing supply.

How does this optimization problem relate to the conditions of the competitive equilibrium? Proof that an optimal solution to (OP) fulfills the competitive equilibrium conditions is e.g. by contradiction, an outline is as follows: Suppose P^* is the market price from the solution of the optimization problem. Now, if there is a buyer i, whose valuation is higher than P^* but does not receive the good, i.e. $d_i = 0$, then you can find a seller with a cost p_j at or just above P^* who can serve at least a small part of that load. But if that is possible, then the value of the objective function increases, contradicting the claim of optimality. Similar argument applies for the supply. Finally, the balance constraint will guarantee that the third condition of a competitive equilibrium is fulfilled in any feasible solution. Proof of equivalence to the other direction is left as an exercise.[3]

We can now proceed after having established that the market price P^* that results from the optimization problem (OP) indeed fulfills the competitive market equilibrium conditions. Given the price P^*, we can reformulate the objective function in the maximization problem (OP) as follows

$$\begin{aligned}
&\max_{d_i,s_j} \sum_i p_i d_i - \sum_j p_j s_j \\
\Leftrightarrow &\max_{d_i,s_j} \sum_i p_i d_i - P^* d_t + P^* s_t - \sum_j p_j s_j \\
\Leftrightarrow &\max_{d_i,s_j} \sum_i p_i d_i - \sum_i P^* d_i + \sum_j P^* s_j - \sum_j p_j s_j \\
\Leftrightarrow &\max_{d_i,s_j} \sum_i (p_i - P^*) d_i + \sum_j (P^* - p_j) s_j
\end{aligned}$$

Here we again use the fact that $d_t = s_t$. But now the last formulation is exactly equivalent to the surplus maximization. The first sum is by definition consumer surplus and the last sum is by definition the producer surplus.

So indeed, based on such theoretical argument, the electricity market equilibrium calculation can be said to be good for the society. This is an idea dating back to Samuelson (1952): the equilibrium conditions of a competitive partial equilibrium are equivalent to the maximization of consumers' and producers' surplus, given the constraint of supply meeting demand[4]. This observation is the basis of electricity market price computation in

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[3] Note that in general there needs also to be a tie-breaking rule e.g. to resolve situations where many bidders have equal prices, this is not important for the conceptual insights.

[4] We slightly misuse the terminology. Of course, the actual tariffs paid by the consumers will be greater as they include all the taxes, levies, and grid charges, and the retailer's margin that are ignored here.

exchanges all over the world (see e.g. Fabra, von der Fehr and Harbord, 2006).

3.2 Optimal producer bids

How do the firms come up with bids in the market place? In the day-ahead market, a producer j needs to commit to a production schedule for each hour in advance. We look at the market of one single hour, as described above, where the producer submits a bid that consists of a price–quantity pair $(p_j,\, s_j)$. The objective of the producer is to maximize the profit from one single plant. We assume that the producer places the bid *as if* the market is competitive, i.e. not taking into account the possibility that the bid affects market outcomes. This would be almost true for a small producer and a sufficient condition for the market to be competitive.

Let's step back to see the usual logic of arriving at the profit maximizing bids. In the standard theory of the firm in microeconomics, the profit of the firm, $\pi(x)$, is given by

$$\pi(x) = pf(x) - \sum_{n=1}^{N} w_n x_n,$$

where x is a vector of inputs, p is the price that the firm receives for its output, $f(x)$ describes the production technology of the firm, $x = (x_1, \ldots, x_N)$ are the inputs required for production, and w_n are the prices of those inputs. Given an output price p, the firm maximize its profit by selecting the use of its inputs x:

$$\max_{(x=x_1,\ldots,x_N)\geq 0} pf(x) - \sum_{n=1}^{N} w_n x_n,$$

from which the optimal use of inputs is readily given by the the first-order conditions:

$$p\frac{\partial f}{\partial x_n} - w_n = 0 \quad \forall n,$$

i.e. the firm should choose a level of production so that the marginal cost, accounting for all inputs, equals marginal revenue. However, as the output is constrained, typically from 0 to the nameplate production capacity K_j of the plant, in general one needs to consider the optimal output given the constraints.

Example 1. Assume the following simplistic production function for a coal plant:

$$f(x_c) = \eta_c x_c,$$

i.e. the quantity of production, $q = f(x_c)$, results from the conversion of coal to electric energy with the power plant that has an electric efficiency of η_c (e.g. 39.50%). We assume away any other costs related to ramp-up and ramp-down, other inputs etc. The plant has a production capacity, K_j. The marginal cost of production is assumed to be constant here so the optimal production decision will be the same at any capacity. There are three cases to go through that will depend on the relation between the marginal cost w_c/η_c and marginal revenue p:

1. The marginal cost is below the market price: produce at full capacity.
2. The marginal cost is above the market price: don't produce at all.
3. The marginal cost is exactly at the market price level: profit equal to zero regardless of the amount produced.

How can the producer realize such a production schedule? By submitting a bid where the bid price p_j is set at the level of marginal costs w_c/η_c and the quantity s_j is set at the capacity limit K_j (feel free to verify the cases).

In addition, and somewhat atypically compared to other markets, the firm may face costs from not producing. Such costs may rise from e.g. the inability of the plant to change its production from one hour to the next, the linkage between production decisions in a hydropower system in the same river, or subsidies that are directly linked to the produced quantity. In choosing the bid price p_j, the bidder needs to consider the marginal cost of production and these *opportunity costs* of not producing.

Example 2. Consider now a nuclear power plant that has a production capacity K_j. We assume that the plant has a relatively small variable cost if it is running, say c. Our current nuclear power plants cannot technically be (safely) turned on and off for a single hour, so shutting down the plant has a relatively high cost $C \gg c$.[5] The profit of the firm is thus given by

$$\pi(x) = \begin{cases} px - cx, & \text{if } x > 0, \\ -C, & \text{otherwise.} \end{cases}$$

If the nuclear plant would follow the same logic as the coal plant, then it would place the bid at $(p_j = c, s_j = K_j)$. However, this logic relies (implicitly!) on the assumption that the cost of not producing is 0, as we assumed for the coal plant. If the realized market price is lower than c, the nuclear plant faces a cost of C. Considering the single hour market only, the nuclear plant would thus want to place a bid $-C$ to avoid the cost of shutting down. In practice, the market places have a technical floor price which is greater than $-C$, so the nuclear plant bids are at the price floor level.

4 Deviations from perfect competition

Everything in the previous section works day to day in dozens of electricity markets around the world. Obviously this does not mean that the markets are perfectly competitive. The key assumption needed is that the bidders would actually submit their bids freely on the basis of their actual valuations. We also need to assume that there are no other distortions.

Strategic behavior

Any deviation from perfectly competitive bidding behavior means that we can no longer guarantee that the objective function is maximizing the consumer and producer surplus. For example, take a perfectly competitive market equilibrium (P^*, Q^*). Now if a supplier bids a generation unit that has a lower cost than P^* with some $P^* + m$, where m is chosen so that in the resulting new equilibrium market quantity Q' is lower than Q^*, then the resulting allocation is no longer societally efficient.

We will not discuss the implications of strategic behavior in any great detail here. While such considerations are important from the point of view of competition authorities, our focus in this course is going to be more on the externalities that the trade in one market causes to the environment.

Price caps and floors

In all electricity markets, prices are capped from above and there is a price floor that limits how low the prices can go. This is in part done to limit the incentives for strategic behavior. If all units are called into operation, the demand may exceed the capacity, and firms can ask very high prices (see Green, 2005). Capping the maximum market price limits the profits. Firm may also strategically manipulate the available capacity to be offered, and a price cap limits these incentives.

However, setting up price limits is in itself a deviation from the assumptions of free competition and may distort the market outcomes. In particular a price cap can limit the scarcity rents that a generation plant otherwise would get from the market. This leads to “missing money” problem: the market prices do not give enough support for new investments, potentially endangering security of supply and transition to the clean energy system. Because of such concerns almost all markets have some additional capacity mechanisms in place to ensure that sufficient capacity is in place to cover all demand levels (Fabra, 2018 provides a comprehensive and accessible overview). However, the price caps can also be efficient in the often typical case where consumers have fixed price

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[5] It could also be the case that the producer chooses to leave the excess balance to the real-time markets, but with again potentially much higher costs for the system and the producers. Throughout we discuss little of these real-time markets as the institutional detail increases and the economic and environmental importance decreases rapidly with them.

tariffs. In such circumstances it may be optimal to ration their consumption (see Joskow and Tirole, 2007).

Subsidies

Also any subsidies, for example to renewable energy, will distort the market outcomes. For example, start again with a perfectly competitive market equilibrium (P^*, Q^*). Now take a supplier who has a generation unit with a higher cost than the market price, $p_j > P^*$, but the unit receives a subsidy s so that the supplier can submit a bid $p_j - s < P^*$. All else equal, such subsidy will mean that either the new equilibrium price is lower, leading to too much consumption, or that a more efficient power plant is superseded by the subsidized plant, or both. In any case, the resulting allocation is no longer societally efficient.

5 Analytical tools

Electricity markets are a rare example where the bids from market participants are collected in a way that supports a micro founded analysis of the market equilibria. Often there is less data available and almost always when going through the theory it is useful to use approximations for the shapes of the supply and demand curves, although the results will often apply in much more generality. For example, we will be using a linear demand functions of the type $Q^D(P) = a - bP$ and supply $Q^S(P) = c + dP$.

6 Further readings

The economics of electricity markets are expressed in a way that makes sense in e.g. Wilson (2002) or Green (2005). For the purposes of a basic course such as this one, the more recent work tends to go in to too much detail and provide little explanation of what now is seen as established groundwork. Some exceptions can certainly be found. For example nice exposition for some historical perspective from the U.S. is found in Borenstein and Bushnell (2015) and Cramton (2017) discusses the impact that renewables can have on the market design.

We will certainly have more material for the more in-depth topics later on in the course.

Exercises (voluntary!)

1. Take the linear demand, $Q^D(P) = a - bP$ and supply $Q^S(P) = c + dP$ equations. Solve the the market equilibrium price.
2. Draw a supply–demand equilibrium for some good. How can you use your drawing to motivate the fact that the shadow price of the balance constraint gives the market price.
3. Take the conditions for a competitive equilibrium. Derive the maximization problem (OP) that is used to solve the market equilibrium from those conditions.
4. How would one include transmission constraints between different zones or nodes to the optimization problem (OP)? (Samuelson's original question was actually about international trade!)

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A Examples of institutions

California

In all power markets, in addition to matching demand with supply, system operators need ancillary services to secure sufficient system reservers for various contingencies at all times. The electricity markets in the U.S. are organized typically around centralized dispatching of generation assets. This means that the market operator collects bids from the market participants and then optimizes the use of the generation fleet to meet the load.

As a prime example of an U.S. system, CAISO in California organizes the procurement of ancillary services simultaneously with the clearing of the energy markets. Each bidder in CAISO can offer the same resource to the day-ahead energy market and the ancillary services markets at the same time. The computation algorithm employed by the operator will then allocate the resources to various markets in the *Integrated Forward Market* so that the total welfare is maximized given the relevant system security and feasibility constraints[6]. As a result, each bidder is awarded the schedule that maximizes the value of their bids in all markets, again subject to the system-wide constraints.

The main ancillary service market types in CAISO are: *Regulation Up*, *Regulation Down*, *Spinning Reserve* and *Non-Spinning Reserve*. Because the system reserves must be available in the case they are needed, the same resource can commit only to one market at the same time. If a resource is committed to any of the ancillary services, then the same resource is not included in the energy market bidding[7].

In addition to the possibility of bidding to several markets, generators can bid alternative schedules to the energy market. For example, a plant can be offered for the whole day at full capacity if it receives a certain price in one schedule or, alternatively, the same plant can be offered only for the peak hours but with a higher bid price. Again, the algorithm by CAISO chooses the schedule that maximizes welfare given the system constraints.

In CAISO bids are submitted to nodes: a *locational marginal price* is calculated for each of the around 3,000 nodes for each hour taking into account the transmission constraints between the nodes. The nodal system in the U.S. is much more granular than the zonal system in use in Europe. The likely historical explanation is that the electricity grid in the relatively sparsely populated U.S. has been much weaker than the grid in Europe. A more refined market model will result in prices reflecting the transmission costs between the nodes, giving the market parties more refined signals for investments.

Finally, CAISO offers a variety of different bidding options. The participants can bid *economic bids*, i.e. price–quantity pairs, or they can choose to *self-schedule* in which case there is no price attached to the quantity. In addition to the physical bids described above, market participants can submit financial *convergence bids*, often called as *virtual bids.* They are included in the determination of the day-ahead market prices in the integrated market price calculation.

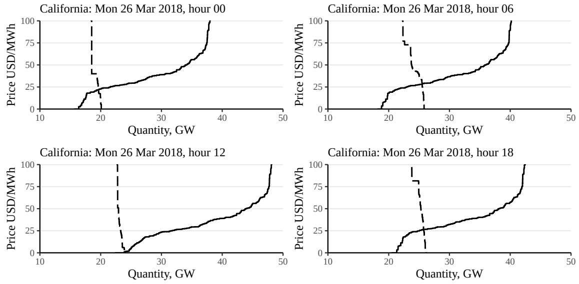

Figure 1: Bid curves for four hours during a day in California.

Nord Pool

In the Nordic day-ahead electricity markets, the market operator, Nord Pool, runs an energy only exchange where hourly supply and demand bids are matched. The bids in the

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[6] The model (OP) is a simplified version.

[7] CAISO applies slightly more accurate method where the same resources can be partially allocated to different markets as long as the capacity of the resource in total is not exceeded.

Nordic area are geographically attached to one of the 12 price areas[8]. Nord Pool carries out two separate rounds of matching: One for each price area and another for a system-wide reference price, the *system price*. In the system price calculation all bids from all price areas, regardless of their location, are combined to a single supply curve and single demand curve that are matched. The data for the area price calculations is not available nor is information on individual bidders, but Nord Pool does report the demand and supply curves that are used in the system price calculation.

In addition to the simple bids for one hour, market participants have the possibility to offer *block bids* that combine a fixed quantity and price over a period of several hours (for the reasons discussed above). The sum of the quantities of the accepted block bids on both sides of the market is reported by the exchange. In addition, Nord Pool reports the total net flow from the surrounding regions through transmission interconnections (interties).

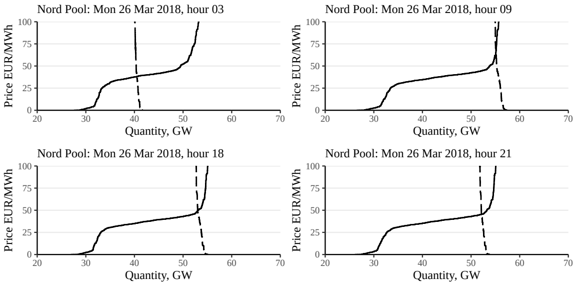

Figure 2: Bid curves for four hours during a day in the Nordic market.

Spain

The logic of European day-ahead electricity markets is mostly unified, and even the market clearing is done simultaneous with the same clearing algorithm (EUPHEMIA) for 25 countries. But like in Nord Pool, the Spanish market operator, OMIE, has special rules to account for e.g. ramping-up and down restrictions.

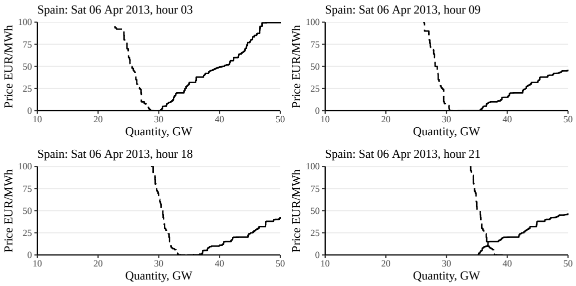

Figure 3: Bid curves for four hours during a day in Spain.

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[8] In the Nordic countries in 2019.